## 2020

## COMPUTER SCIENCE - HONOURS

Paper: DSE-B-1
(Operation Research)
Full Marks: 50
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer question number $\mathbf{1}$ and any four questions from the rest.

1. Answer any five questions:
(a) What do you understand by initial basic feasible solution?
(b) How is the optimality of simplex method checked?
(c) Define Saddle Point.
(d) Convert the following inequations to an equation.

$$
\begin{aligned}
& 3 x_{1}+2 x_{2} \leq 50 \\
& 3 x_{1}-x_{2} \geq 100
\end{aligned}
$$

(e) When is artificial variable used in an LPP?
(f) Define linear programing problem (LPP).
(g) Differentiate between transportation problem and assignment problem.
(h) State the basic objectives of CPM and PERT technique.
2. (a) Use simplex method to solve the following linear programing problem.

$$
\begin{aligned}
& \text { Maximize } \mathrm{Z}=x_{1}+2 x_{2} \\
& \text { Subject to : }-x_{1}+2 x_{2} \leq 8 \\
& x_{1}+2 x_{2} \leq 12 \\
& x_{1}-2 x_{2} \leq 3 ; \\
& x_{1} \geq 0, \quad x_{2} \geq 0,
\end{aligned}
$$

(b) Find the maximum value of

$$
\begin{array}{ll}
\mathrm{z}=4 \mathrm{x}+\mathrm{y} \\
& \\
\text { Subject to, } & x+y \leq 50 \\
& 3 x+y \leq 90 \\
& x, y \geq 0
\end{array}
$$

(using Graphical Solution method)
3. (a) Prove that the dual of the dual of an LPP is the primal.
(b) Obtain the dual problem for the following LPP

Maximize $f(x)=2 x_{1}+5 x_{2}+6 x_{3}$
Subject to the constraints :
$5 x_{1}+6 x_{2}-x_{3} \leq 3$
$-2 x_{1}+x_{2}+4 x_{3} \leq 4$
$x_{1}-5 x_{2}+3 x_{3} \leq 1$
$-3 x_{1}-3 x_{2}+7 x_{3} \leq 6$
$x_{1}, x_{2}, x_{3} \geq 0$.
4. (a) State the problem of transportation mathematically.
(b) Solve the following transportation problem to minimize the total cost, obtaining the initial basic feasible solution by North West corner rule.

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{D}_{4}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 6 | 4 | 1 | 5 | 14 |
| $\mathrm{O}_{2}$ | 8 | 9 | 2 | 7 | 16 |
| $\mathrm{O}_{3}$ | 4 | 3 | 6 | 2 | 5 |
|  | 6 | 10 | 15 | 4 | 35 |

where $\mathrm{O}_{i}$ and $\mathrm{D}_{j} \mathrm{~s}$ represent the $i^{\text {th }}$ origin and $j^{\text {th }}$ destination respectively.
Find the optimal solution.
5. (a) A department head has four tasks, to be performed and three subordinates, the subordinates differ in efficiency. The estimates of times, each subordinate would take to perform is given below in the matrix. How should he allocate the tasks one to each man, so as to minimize the total man-hours?

|  | Men |  |  |
| :---: | :---: | :---: | :---: |
| Task | 1 | 2 | 3 |
| I | 9 | 26 | 15 |
| II | 13 | 27 | 6 |
| III | 35 | 20 | 15 |
| IV | 18 | 30 | 20 |

(b) Prove that the optimal solution of an assignment problem is unchanged if we add or subtract the same constant to the entries of any row or column of the cost matrix.
6. For the game with pay-off matrix :

$$
\begin{gathered}
B_{1} B_{2} \\
A_{1} \\
A_{2}
\end{gathered}\left[\begin{array}{cc}
10 & 6 \\
8 & 2
\end{array}\right]
$$

determine the best strategies for players A and B and also the values of the game for them. Is this game (i) fair (ii) strictly determinable?
7. (a) Find the critical path for the project illustrated in the network below :

(b) Give a procedure for resource levelling using PERT / CPM.
8. (a) Obtain the dual problem of the full LPP.

$$
\begin{array}{ll}
\operatorname{Max} & Z= \\
\text { S.t. } & x_{1}-2 x_{2}+3 x_{3} \\
& -2 x_{1}+x_{2}+3 x_{3}=2 \\
& 2 x_{1}+3 x_{2}+4 x_{3}=1 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

(b) Find all the basic feasible solutions of the equations
$2 x_{1}+6 x_{2}+2 x_{3}+x_{4}=3$

$$
6 x_{1}+4 x_{2}+4 x_{3}+6 x_{4}=2
$$

