

2020

PHYSICS — HONOURS

Paper : CC-2

(Mechanics)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any four** questions from the rest.

1. Answer **any five** questions of the following : 2×5
- Show that mutually interacting forces on a system of particles have no effect on its total linear momentum.
 - A solid sphere and a solid cylinder having same mass and same radii roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.
 - 'In streamline flow of a Newtonian fluid two streamlines never intersect'— Explain.
 - Prove that the areal velocity of a particle moving under a central force field is constant.
 - What is the rotational period of a binary star consisting of two equal masses, M and separated by distance L ?
 - Find the degrees of freedom of a system of two point masses joined by a massless rigid rod in a 3-dimensional space.
2. (a) A particle is moving in a plane in such a way that its polar co-ordinates are given by $r = 2t + 3$ and $\theta = 3t - t^2$. Obtain the radial and transverse components of instantaneous acceleration.
- (b) A particle of mass ' m ' at rest at $(a, 0, 0)$ subjected to a force $\vec{F} = -\frac{k}{x^3} \hat{x}$, where k is a positive constant. Find the time taken by the particle to reach the origin.
- (c) Given $\vec{F} = -r\hat{r}$ is a conservative force field. Find the corresponding scalar potential. 4+4+2
3. A particle of mass m moves along a trajectory given by $x = x_0 \cos \omega_1 t$, $y = y_0 \sin \omega_2 t$, where x_0 and y_0 are constants.
- Find the x and y components of the force. What is the condition under which the force is a central one?
 - Find the potential energy as a function of x and y .
 - Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved. (2+1)+3+(2+2)

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4. (a) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.
- (b) Prove that total energy of a particle of mass 'm' acted upon by a central force is given by,

$$E = \frac{L^2}{2m} \left[u^2 + \left(\frac{du}{d\theta} \right)^2 \right] + V(r)$$

where L is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$, r and θ being the polar co-ordinates. 5+5

5. (a) Show how a fictitious force arises in a non-inertial frame which is moving with a constant acceleration in a given direction with respect to a fixed frame.
- (b) Let S' be a reference frame which is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector \vec{A} ,

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refer to time derivatives with respect to S and S' frames, respectively.

- (c) Two reference frames, one is fixed and other one is rotating, have common origin. Obtain the equation of motion of a particle of mass 'm' with respect to the rotating frame. Discuss about the different fictitious forces arise in the rotating frame. 2+4+4
6. (a) Show that the angular momentum vector \vec{L} is not always along the same direction as the instantaneous axis of rotation.
- (b) Determine the moment of inertia tensor for the configuration in which four point masses of 1, 2, 3 and 4 units are located at (1, 0, 0), (1, 1, 0), (1, 1, 1) and (1, 1, -1) units, respectively.
- (c) A rigid body is rotating under the influence of an external torque $\vec{N}^{(e)}$. If the angular velocity is $\vec{\omega}$ and kinetic energy is T , show that

$$\frac{dT}{dt} = \vec{N}^{(e)} \cdot \vec{\omega}$$

when the axes of the body co-ordinates are taken as principal axes.

- (d) Indicate the principal axes for a homogeneous sphere and a cylinder in neatly labelled sketches. 2+3+3+2
7. (a) Set up Euler's equation for an incompressible fluid and establish Bernoulli's equation of fluid motion stating the assumptions used. 6

(3)

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- (b) A pipe of varying diameter is used to lift water by 7m. The area of cross-section of the pipe at the base is 125 cm^2 and the pressure here is $2.5 \times 10^5 \text{ Pa}$. The area of cross-section of the pipe at the top is 25 cm^2 . The rate of flow of water is $3 \times 10^{-2} \text{ m}^3/\text{sec}$. Calculate the pressure of water at the top, neglecting energy losses. 4

Or,

A copper wire of diameter 1 mm . and length 3 meters has Young's modulus $12.5 \times 10^{11} \text{ dynes per sq.cm.}$, If a weight of 10 kg . is attached to one end, what extension is produced? If the Poisson's ratio is 0.26, what lateral compression is produced? 4
