2021

MATHEMATICS — HONOURS

Paper : CC-10

(Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings unless otherwise stated.

1. Answer the following multiple choice questions with only one correct option. Choose the correct option and justify : (1+1)×10

(a) Let ABC be a triangle. Three forces $\vec{P}, \vec{Q}, \vec{R}$ act along $\overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{AB}$, respectively, and their resultant passes through the in-centre of the triangle. Then

(i) $2\vec{P}+\vec{Q}+\vec{R}=\vec{0}$ (ii) $\vec{P}+2\vec{Q}+\vec{R}=\vec{0}$

(iii) $\vec{P}+\vec{Q}+2\vec{R}=\vec{0}$ (iv) none of these.

(b) Suppose a system of non-coplanar forces $\vec{F_i}\,(i=1, 2, ..., n)$ acting on a rigid body is initially reduced to a single force $\vec{R}$ acting at an arbitrarily chosen point O (of the body) together with a couple $\vec{G}$. If the system is further reduced to a wrench, then its **pitch** is given by

(i) $p=\vec{G}.\vec{R}$ (ii) $p=\dfrac{\vec{G}.\vec{R}}{R}$

(iii) $p=\dfrac{\vec{G}.\vec{R}}{R^2}$ (iv) $p=\dfrac{\vec{G}.\vec{R}}{R^3}$

where $R=|\vec{R}|$.

(c) The moments of a system of coplanar forces about the points (0, 0), (a, 0), (0, a) are respectively $a\omega$, $2a\omega$, $3a\omega$. Then the components of their resultant parallel to the coordinate axes are :

(i) $X = -2\omega,\ Y = -\omega,\ G = a\omega$ (ii) $X = 2\omega,\ Y = \omega,\ G = -a\omega$

(iii) $X = 2\omega,\ Y = -\omega,\ G = a\omega$ (iv) $X = -2\omega,\ Y = \omega,\ G = -a\omega$.

(d) A uniform cubical box of edge d is placed on the top of a fixed sphere of radius r. The equilibrium of the box will be stable if

(i) $d > r$ (ii) $d < r$

(iii) $d < 2r$ (iv) $d > 2r$.

(e) A particle describes a curve, whose polar equation is $r = 6e^{\theta}$. If the angular velocity is constant, the transverse acceleration varies as

(i) r (ii) r^2

(iii) r^3 (iv) r^4.

(f) A particle moves in a plane curve so that its tangential acceleration is constant and the magnitudes of the tangential velocity and normal acceleration are in a constant ratio. The intrinsic equation of the curve is

(i) $s = A\tan\psi + B\psi + C$ (ii) $s = A\psi^3 + B\psi + C$

(iii) $s = A\psi^2 + B\psi + C$ (iv) $s = A\sin\psi + B\cos\psi + C$

where A, B, C are constants.

(g) A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. The gun recoils with a velocity

(i) $\sqrt{\dfrac{mE}{M+m}}$ (ii) $\sqrt{\dfrac{mE}{M(M+m)}}$

(iii) $\sqrt{\dfrac{2mE}{M(M+m)}}$ (iv) $\sqrt{\dfrac{2mE}{M+m}}$.

(h) A particle moving along a straight line has the relation $v^2 = 4 - x^2$ between the velocity v and displacement x from origin O at any time t, then the motion is

(i) S.H.M. with period $\dfrac{\pi}{2}$ (ii) S.H.M. with period π

(iii) S.H.M. with period 2π (iv) none of these.

(i) A particle describes a parabola with uniform speed. Then the angular velocity of the particle about the focus S, at any point P, varies

(i) inversely as $(SP)^{\frac{3}{2}}$ (ii) inversely as $(SP)^{\frac{2}{3}}$

(iii) inversely as $(SP)^{-\frac{3}{2}}$ (iv) none of these.

(j) A circular plate rotates about an axis through its centre perpendicular to its plane with angular velocity ω. This axis is set free and a point in the circumference of the plate is fixed. The resulting angular velocity is

(i) $\dfrac{\omega}{2}$ (ii) $\dfrac{2}{\omega}$

(iii) $\dfrac{3}{\omega}$ (iv) $\dfrac{\omega}{3}$.

Unit - 1

(Marks : 05)

2. Answer ***any one*** question : 5×1

(a) If all the forces in a coplanar system are rotated about their points of application in the same plane through the same angle in the same sense, prove that their resultant passes through a fixed point in the plane.

(b) A force parallel to the axis of z acts at the point $(a, 0, 0)$ and an equal force perpendicular to the axis of z acts at the point $(-a, 0, 0)$. Show that the central axis of the system lies on the surface

$$z^2\left(x^2+y^2\right)=\left(x^2+y^2-ax\right)^2.$$

Unit - 2

(Marks : 05)

3. Answer ***any one*** question : 5×1

(a) A rod AB of weight W is movable about the end A and the end B is attached to a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. Prove, by the principle of virtual work, that the horizontal force necessary to keep the ring at rest is $\dfrac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)}$, where α, β are respectively the inclinations of the rod and the string to the horizontal.

(b) A square lamina rests with its plane perpendicular to a smooth wall, one corner being attached to a point in the wall by a fine string of length equal to the side of the square. Find the position of equilibrium and show that it is stable.

Unit - 3

(Marks : 10)

4. Answer ***any two*** questions : 5×2

(a) A particle is moving in a plane under the action of an attracting force to a fixed point in the plane, equal to μ times the distance from that point per unit mass. The initial coordinates and velocity components with respect to fixed rectangular axes passing through the centre of force are (a, b) and (U, V), respectively. Find the position of the particle at time t and show that the path described by the particle is $\mu(bx-ay)^2+(Uy-Vx)^2=(bU-aV)^2$.

(b) A particle moves in the curve $y = a \log_e \sec(x/a)$ in such a way that tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature.

(c) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the recoil velocity of the gun is $\sqrt{\dfrac{2m^2gh}{M(M+m)}}$.

(d) A uniform chain of length $2a$ is hung over a smooth peg so that the length of it on the two sides are $l - c$ and $l + c$. Motion is allowed to ensue. Use principle of energy to show that the chain leaves the peg after a time : $\sqrt{\dfrac{l}{g}}\log_e\left(\dfrac{l+\sqrt{l^2-c^2}}{c}\right)$.

Unit - 4

(Marks : 15)

5. Answer ***any one*** question : 7×1

(a) A particle is projected with velocity u at an inclination α (acute) above the horizon in a medium that resists the motion by a force kv per unit mass, where v is the velocity of the particle. Prove that the equation of the trajectory is $y = \dfrac{g}{k^2}\log_e\left(1 - \dfrac{kx}{u\cos\alpha}\right) + \dfrac{x}{u\cos\alpha}\left(u\sin\alpha + \dfrac{g}{k}\right)$, where x and y are respectively the horizontal and vertical (upward) coordinates of the particle measured from the point of projection.

(b) A mass m hangs from a fixed point by a light string and is given a small vertical displacement. If l be the length of the string when the system is in equilibrium and n be the number of oscillations per second then show that the natural length of the string is $l - \dfrac{g}{4\pi^2 n^2}$.

6. Answer ***any one*** question :

(a) A particle describes a path which is nearly a circle about a centre of force under the action of a central force $\phi(u)$ at its centre. Find the condition that the motion may be stable. Also find the apsidal angle in this case. 6+2

(b) A particle, of mass m, is projected vertically under gravity with a velocity λV, the resistance of the air being mk times the velocity; show that the greatest height attained by the particle is $\dfrac{V^2}{g}[\lambda - \log(1+\lambda)]$ and after a time, $\dfrac{V}{g}\log(1+\lambda)$, where V is the terminal velocity of the particle. 4+4

Unit - 5

(Marks : 10)

7. Answer ***any one*** question :

(a) (i) Define *linear momentum* of a multi-particle system. Show that in any motion of a multi-particle system, the rate of increase of its linear momentum is equal to the total external forces acting on it.

(ii) Using linear momentum principle, explain the basic principle of *rocket propulsion*.

(iii) Define angular momentum of 'a system of particles' about a point.

The position of a particle P of mass m at time t is given by $x = a\theta^2$, $y = 2a\theta$, $z = 0$, where $\theta = \theta(t)$. Find the angular momentum of P about the point $A(a, 0, 0)$. (1+3)+2+(1+3)

(b) (i) Explain Newton's experimental law on collision.

(ii) Two smooth spheres of masses m_1 and m_2 impinge on one another with speeds u_1 and u_2, at angles α_1, α_2, respectively, with the line of impact. Let e be the coefficient of restitution. Find the magnitude and direction of the velocities of the spheres just after the impact. Also find the impulse due to the impact and the loss of kinetic energy. 2+[4+(2+2)]