2021

PHYSICS — HONOURS

(2019–20 Syllabus)

Paper : CC-10

(Quantum Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) How many lines of spots on the detecting screen will be produced if Stern-Gerlach experiment is performed with an atom of total angular momentum J ?

(b) The wave function of a particle of mass m in one-dimensional potential $V(x) = \frac{1}{2} m\omega^2 x^2$ has the form $\psi(x) = Ae^{-\frac{\alpha x^2}{2}}$ in ground state, where A is a normalization constant and α is a positive constant. Making use of Schrödinger equation, find the ground state energy E of the particle.

(c) In the ground state of harmonic oscillator, calculate the probability of finding the particle outside the classically allowed region.

[You may use the result $erf(1) = \frac{2}{\sqrt{\pi}} \int_0^1 e^{-x^2} dx = 0.8427$]

(d) 'A free particle does not has definite energy'– Explain.

(e) Evaluate $\left[\hat{L}_x^{\,2} + \hat{L}_y^{\,2},\ \hat{L}_z^{\,2}\right]$ where $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ are components or orbital angular momentum operator.

(f) If Parity operator $\hat{P}$ satisfies $\hat{P}\,\Psi(x) = \Psi(-x)$, show that $\hat{P}$ has only two eigenvalues 1 and –1. Find the eigenfunction for each of them.

(g) Show that for all the inert gases term symbol is $^1S_0$.

2. A particle of mass m is confined in a potential :

$$V(x) = \begin{cases} \frac{1}{2} m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

(a) Using the energy eigenfunction $\Psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right) e^{-\frac{m\omega}{2\hbar}x^2}$ and energy eigenvalue $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ of Harmonic oscillator, find the energy eigenfunction and energy eigenvalue of the particle in the n^{th} stationary state.

(b) Draw the ground state and first exited state wave functions along with the potential of the particle.

(c) The particle in the above potential starts out in the state $\Psi(x) = -\frac{1}{\sqrt{5}}\Psi_0 + \frac{2}{\sqrt{5}}\Psi_1$, where Ψ_0 and Ψ_1 are ground state and first excited state of the particle respectively. Calculate the energy expectation value. 4+3+3

3. At time $t = 0$, a free particle is described by the following Gaussian wave function

$$\psi(x) = Ae^{-\frac{x^2}{2\sigma_0^2} + \frac{i}{\hbar}p_0 x},$$

where A is a constant and other symbols have their usual meanings.

(a) Normalize the wave function.

(b) Find the wave function in momentum space.

(c) Hence calculate $< p >$ and $< p^2 >$ in momentum space. 2+4+4

4. The normalized wave function for the ground state of hydrogen like atom is

$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{\frac{3}{2}} e^{-\frac{Zr}{a_0}}$$, where a_0 is the Bohr radius.

(a) Calculate the most probable distance.

(b) Calculate the average distance of the electron from the nucleus.

(c) Sketch the radial probability distribution function $P_{100}(r)$.

(d) Calculate the average value of modulus of the Coulomb force acting on the electron. 3+2+2+3

5. (a) Consider $\Psi(\theta, \phi) = A\,[Y_{1,-1} + Y_{1,1}]$ where $Y_{l,m}$ are spherical harmonics. Find

(i) A

(ii) Is $\Psi(\theta, \phi)$ eigenfunction of $\hat{L}^2$?

(iii) Is $\Psi(\theta, \phi)$ eigenfunction of $\hat{L}_z$?

(iv) Calculate $< L^2 >$ and $< L_z >$ for the state $\Psi(\theta, \phi)$.

(b) $\alpha_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \left| \frac{1}{2}\ \frac{1}{2} \right\rangle$ and $\alpha_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \left| \frac{1}{2}\ \left(-\frac{1}{2}\right) \right\rangle$ are two eigenstates of a spin $\frac{1}{2}$ particle. The x component of the spin operator S is given by $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(i) Find the normalizes eigenstates and eigenvalues of S_x.

(ii) Express any general state of the spin $\frac{1}{2}$ particle $\alpha = \begin{pmatrix} a \\ b \end{pmatrix}$ as a linear combination of the eigenstates of S_x.

(1+½+½+1½+1½)+(3+2)

6. (a) $|\alpha\rangle$ and $|\beta\rangle$ are two states of a spin $\frac{1}{2}$ particle. Obtain the normalized triplet and singlet spin states formed by two spin $\frac{1}{2}$ particles.

(b) Consider the finite square well potential $V(x) = \begin{cases} -V_0 & \text{for} -a<x<a \\ 0 & \text{for}\ |x|>a \end{cases}$

where V_0 is a positive constant and $2a$ is the width of the potential well.

(i) Derive the transcendental equation determining the discrete energy eigenvalues for symmetric wave functions (bound states).

(ii) Find the energy eigenvalues for the symmetric wave functions when the potential well is deep and wide.

4+(4+2)

Please Turn Over

7. (a) Consider the Schrödinger equation :

$$\left(-\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V\right)\Psi = E\Psi$$

for a two-particle system where potential V is a function of $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Show that above equation can be written as

$$\left[-\frac{\hbar^2}{2M}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + V(\vec{r})\right]\Psi = E\Psi,$$

where $M = m_1 + m_2$; $\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{M}$, $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\mu = \frac{m_1 m_2}{m_1 + m_2}$.

(b) Consider the spin-orbit correction term $H_{SO}^1 = K\,\vec{S}\cdot\vec{L}$ where K is a constant.

Show that H_{SO}^1 commutes with L^2, S^2, J^2 and J_z.

(c) Find the Lande g-factor for ${}^2P_{3/2}$. 4+4+2

2021

PHYSICS — HONOURS

(2018-2019 Syllabus)

Paper : CC-10

(Analog System and Applications)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) Compare between the performances of a C-filter and a π-filter.

(b) Explain the working principle of an LED.

(c) What is drift velocity? How is it related to mobility?

(d) What are the advantages of negative feedback?

(e) What is the pinch-off effect in a JFET?

(f) Indicate the lower and upper cut-off frequency on the frequency response curve of a CE-amplifier.

(g) What is slew rate of an OPAMP?

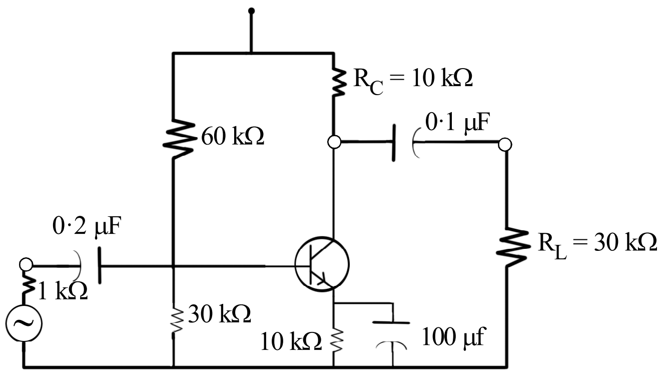
2. (a) If the bandgap of silicon be 1.1 eV, upto what wavelength of light can it absorb?

(b) What is the load line of an active device? How can you specify the endpoints of the load line in a CE transistor circuit?

(c) What are hybrid parameters of a transistor? Why are they named so?

(d) What is an emitter follower? 2+(1+2)+(2+1)+2

3. (a) Find the lower cut-off frequency of a self-biased circuit (with voltage devider) of CE-amplifier given below :



Please Turn Over

(b) Explain the current amplification factors α and β for CB and CE configuration respectively. Obtain the relation between them.

(c) Calculate I_E in a transistor for which $\beta = 50$ and $I_B = 20\ \mu A$. 4+(2+2)+2

4. (a) Explain how a JFET can be used as a voltage controlled current source.

(b) Draw the common source drain characteristics of a JFET and explain the behaviour in different regions.

(c) Show that higher gain of an R-C coupled amplifier offers a reduced bandwidth. 3+(2+3)+2

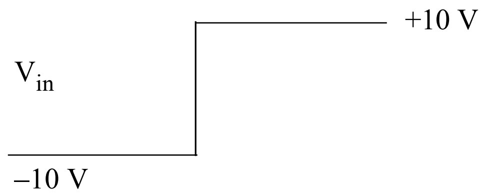
5. (a) What is meant by frequency stability of an oscillator? Draw the circuit diagram of a Hartley oscillator. Find the frequency of oscillation and condition for oscillation.

(b) Write down Barkhausen criterion for oscillation, explaining the terms. (2+2+4)+2

6. (a) The CMRR of a differential amplifier using OPAMP is 100 dB. The output voltage is 2V for a differential input of 200 μV. Determine the common mode gain.

(b) Explain with circuit diagram the action of a zero crossing detector using OPAMP.

(c) Consider the OPAMP integrator with R = 100 kΩ, C = 0·01 μF operated with 250 Hz input voltage. Find the expression for output wave form (V_0).



Input wave form

For the above mentioned input square wave form, draw the output wave form. 2+4+(2+2)

7. (a) What is self bias? Draw the circuit diagram showing the self bias of an *n-p-n* transistor in the CE configuration.

(b) Explain physically how the self biasing resistor improves the stability. Explain the functions of the bypass and the coupling capacitors.

(c) What are the advantages of *h*-parameters? (1+2)+(2+3)+2