2021

PHYSICS — HONOURS

Paper : CC-14

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question number 1*** *and* ***any four*** *from the rest.*

1. Answer ***any five*** questions: 2×5

(a) For a 1D harmonic oscillator with energy E to E+δE draw the phase space diagram. Indicate what is the difference between macrostate and a microstate in the context of this diagram.

(b) Justify if one can use equipartition of energy for a Hamiltonian of the form $H = \alpha p^2 + \beta pq + \gamma q^2$.

(c) In a thermodynamics system in equilibrium each molecule can exist in three possible states with probabilities $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{6}$ respectively. Calculate entropy per molecule.

(d) An energy level ϵ is 3 fold degenerate. In how many ways can two MB particles be distributed over them?

(f) Explain physically why the specific heat of free electron gas (i.e. metals) should vary linearly with temperature.

(g) State two properties of liquid He^4 below the critical point.

2. (a) Assuming Planck's expression for average energy in the energy density spectrum for blackbody radiation.

(i) establish Wien's law for high energy end and

(ii) establish Rayleigh-Jean's law for the low energy end.

(b) What is the chemical potential of the radiation field? Justify your answer.

(c) Write down the expression for the partition function of radiation, assuming it to be a gas of an oscillators each having energy $h\nu$. Hence calculate

(i) free energy for the radiation

(ii) obtain the expression for entropy. (2+2)+2+(1+1+2)

3. (a) A system has two energy levels 0 and ϵ which are g_0 and g_1 fold degenerate respectively. At thermal equilibrium–

(i) write down partition function

(ii) find the average energy

(iii) find the specific heat and its high temperature behaviours.

(b) Consider N mutually independent spin in thermal equilibrium at temperature T. Each spin has two independent states $+\epsilon$ and $-\epsilon$.

(i) Write the partition function.

(ii) Find the expression for free energy.

(iii) Hence, show that this system has a possibility of negative absolute temperature. (1+2+2)+(1+2+2)

4. (a) Consider a system of two particles, each of which can be in any one of three quantum states of respective energies 0, ϵ and 3ϵ. The system is in contact with a heat reservoir at temperature T. Calculate the canonical partition function

(i) if the two particles are distinguishable

(ii) if the two particles are fermions of same type

(iii) if the two particles are Bosons of same type.

(b) Consider two identical particles occupying a hundred fold degenerate energy level. Calculate, the number of microstates for (i) BE, (ii) FD and (iii) MB statistics. (3+2+2)+3

5. (a) What is the basic difference between the canonical and Grand canonical ensemble?

(b) Using canonical ensemble establish that

$$\langle E^2\rangle - \langle E\rangle^2 = K_B T^2 C_V$$

Justify that $\sqrt{\langle E^2\rangle - \langle E\rangle^2}/\langle E\rangle$ tends to zero for an ideal gas in the thermodynamic limit.

(c) For a one dimensional classical harmonic oscillator energy is represented as

$$E = \frac{p^2}{2m} + \frac{1}{2}ma^2x^2$$

Show that $\langle KE\rangle = \langle PE\rangle = \frac{1}{2}K_B T$. 1+(3+2)+4

6. (a) What is meant by a degenerate Fermi Gas?

(b) Draw the F.D distribution function for $T = 0$ and $T \neq 0$.

(c) Taking into account the spin degeneracy factor calculate the single particle density of an electron confined in a two dimensional square of area A. Calculate the Fermi energy.

(d) Establish a relation between the average energy and Fermi energy for a degenerate Fermi gas in 2D as $T \to 0$. 2+2+(2+2)+2

7. (a) An ideal monatomic gas of *N* molecules, each of mass *m*, is in Thermal equilibrium of temperature *T*. The gas is contained in a volume *V*. Calculate the partition function of the system. Also calculate the average energy.

(b) The number of states (Ω) accessable to a system of *N* molecules of ideal gas of energy *E*, confined in volume *V* is of the form $\Omega(N, V, E) \propto V^N f(E)$. From this relation, find the equation of state of ideal gas. (3+2)+5