2022

MATHEMATICS — GENERAL

Paper : SEC-B-1

(Mathematical Logic)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

[Notations have their usual meanings.]

1. Choose the correct option and justify your answer : (1+1)×10

(a) The truth value of $(p \vee q) \wedge (p \vee \sim q)$ depends on the truth value(s) of

(i) p (ii) q

(iii) both p and q (iv) none of these.

(b) If $p \rightarrow q$ is logically equivalent to A, then A may be

(i) $\sim p \wedge q$ (ii) $p \wedge \sim q$

(iii) $\sim p \vee q$ (iv) $p \vee \sim q$.

(c) $\sim(\sim(\sim p)))$ is equivalent to

(i) p (ii) $\sim \sim p$

(iii) tautology (iv) $\sim p$.

(d) If A is a tautology and $A \rightarrow B$ is a tautology, then

(i) B is contradiction (ii) B is a tautology

(iii) B is contingent (iv) $B \rightarrow A$ is tautology.

(e) Which one of the following is in Prenex normal form?

(i) $\forall x(x < y) \rightarrow \exists z(x < z \wedge z < y)$ (ii) $\exists v \sim P \rightarrow \forall v P$

(iii) $\exists v(P \rightarrow Q)$ (iv) $\exists v(P \rightarrow Q) \leftrightarrow (\forall v(Q \rightarrow P))$.

(f) What is the correct translation of the following statement into mathematical logic? 'Some real numbers are rational'

(i) $\exists x$ (real $(x) \vee$ rational (x)) (ii) $\exists x$ (real $(x) \wedge$ rational (x))

(iii) $\forall x$ (real $(x) \rightarrow$ rational (x)) (iv) $\exists x$ (rational $(x) \rightarrow$ real (x)).

(g) Let $P(x)$ be a predicate on a non-empty set D. Then, $\sim \forall x P(x)$ is logically equivalent to

(i) $\exists x \sim P(x)$ (ii) $\sim \exists x\, P(x)$

(iii) $\forall x \sim P(x)$ (iv) $\sim \exists x \neg P(x)$.

(h) Let $P(x)$ be 'x is a teacher' and $Q(x)$ be 'x is a singer' where the universe of discourse is set of all persons. Then, $(\forall x P(x)) \vee (\exists y Q(y))$ is

(i) all persons are teacher and singer

(ii) some are teacher and some are singer

(iii) all men are teacher and some are singer

(iv) none of these.

(i) The rule of inference of any first order theory are

(i) Modus Ponens (ii) Generalizations

(iii) both (i) and (ii) (iv) none of these.

(j) Which of the following statement formulae is not in disjunctive normal form?

(i) $p \vee (\sim q \wedge r)$ (ii) $\sim q \vee q$

(iii) $(p \vee \sim q) \wedge (\sim p \vee q)$ (iv) p.

Unit - I

2. Answer ***any two*** questions :

(a) Find the truth table of the statement formula 5

$(p \to (q \to r)) \to ((p \wedge q) \to r)$

(b) Distinguish between object language and metalanguage with suitable examples. 5

(c) Write notes on :

(i) Logical consequence

(ii) Logical Equivalence

(iii) Statement bundle. 2+2+1

Unit - II

3. Answer ***any six*** questions :

(a) Define an adequate system of connectives. Is $\{\vee, \to\}$ an adequate system of connectives? Justify your answer. 1+4

(b) Determine whether the statement forms are logically equivalent $(A \vee (B \leftrightarrow C))$ and $((A \vee B) \leftrightarrow (A \vee C))$. 5

(c) Find the CNF of $\sim(p \vee q) \leftrightarrow (p \wedge q)$. 5

(d) Determine the validity of the following arguments :

'If two sides of a triangle are equal, then two opposite angles are equal. Two sides of a triangle are not equal. Therefore, the opposite angles are not equal.' 5

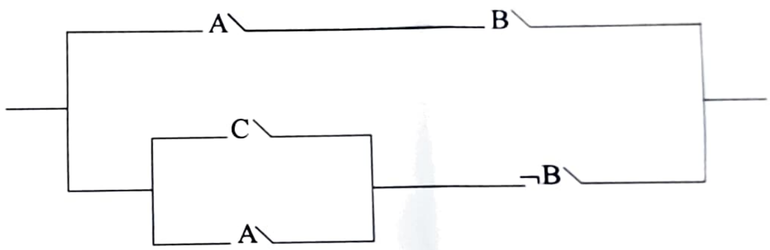
(e) Examine whether the following statement formula is a tautology, contradiction or a contingent

$\sim((p \to q) \leftrightarrow \sim(p \wedge \sim q))$ 5

(f) When is a set of well formed formulae said to be (i) Consistent (ii) maximally consistent?

Also, prove that Σ of well formed formulae is consistent if every finite subset of Σ is consistent. 1+1+3

(g) Find the simpliest equivalent circuit of 5



(h) Prove that every theorem of propositional logic is a tautology. 5

(i) Prove that $\vdash (A \to B) \to ((B \to C) \to (A \to C))$ where A, B and C are well formed formulae in propositional calculus. 5

(j) Show that the following are theorems of propositional logic : 3+2

(i) $\sim \alpha \to (\alpha \to \beta)$

(ii) $\sim \sim \alpha \to \alpha$

where α and β are well formed formulae for Propositional Calculus. 3+2

Unit-III

4. Answer ***any four*** questions :

(a) Let us consider the predicates $P(x)$: x is prime, $Q(x)$: x is even, $R(x, y)$: $x \mid y$, where the universe of discourse is the set $\mathbb{Z}$ of all integers. Translate each of the following into English sentences :

(i) $Q(2) \wedge P(2)$

(ii) $\forall x(R(2, x) \to Q(x))$

(iii) $\exists x(Q(x) \wedge R(x, 6))$ 1+2+2

(b) When is an assertion in Predicate Calculus logically valid? Show whether the following argument is valid :

If x is an odd integer then x^2 is also odd. y is a particular integer that is odd. Therefore y^2 is odd. 1+4

(c) Find the negation of the following statements :

(i) $\forall x(x \leq x^2)$

(ii) $\exists y \forall x(x + y = 0)$. 2+3

(d) Define an interpretation for the language of first-order Predicate logic and the notion of satisfiability with respect to this interpretation. 5

(e) Find the Prenex normal form (PNF) of the following first-order formula :

$\forall x((\exists y\, P(y)) \vee ((\exists z Q(z)) \rightarrow R(x))$. 5

(f) Restore the parentheses to the following :

(i) $(\forall x_2) A_1^1 (x_2) \rightarrow A_1^1 (x_2)$

(ii) $(\forall x_2)(\exists x_1) A_1^2 (x_2 x_1)$

Explain the occurrences (free or bound) of x_1 and x_2 of the wffs given in (i) and (ii) after restoring. 1+1+3

(g) Show that the wff given below is not logically valid

$\left(\forall x_1 A_1^1(x_1) \rightarrow \forall x_1 A_2^1(x_1)\right) \rightarrow \left(\forall x_1 (A_1^1(x_1) \rightarrow A_2^1(x_1)\right)$ 5

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