

2023

COMMERCE

Paper : CC 203

(Operations Research)

Full Marks : 40

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***Module - I**Answer *any two* questions.

1. (a) Mercury Ltd. is planning to invest ₹ 60 Lakh in a portfolio of investments. The following information has been collected about the risk and return patterns of some investments :

Investment	Projected Return
Shares of SB Infotech Ltd.	15.0%
Shares of Samurai Automobiles Ltd.	18.0%
Government Bonds	8.25%
HSFC Money Manager Fund	9.50%
XBI Mutual Fund	16.00%
BTI Mutual Fund	10.00%

Additional information :

- (i) The company wants at least 35% of the investments in Government Bonds.
- (ii) The company has also specified that at least 22% of the investments should be in HSFC Money Manager Fund and the amount invested in shares should not exceed the total amount invested in mutual funds.
- (iii) Due to high volatility in the stock market, a higher perceived risk of the shares is expected by the company. So the company decided that the investment in two shares should not exceed ₹ 10.50 Lakh.
- (iv) The amount invested in XBI Mutual Fund should be no more than the amount invested in BTI Mutual Fund.

As a financial planner of the company, formulate the above as a linear programming problem to decide the amount of money to be invested in each of the alternative investment avenues to obtain the highest annual total return for the company.

Please Turn Over

- (b) For the following pay-off matrix, transform the zero sum game into an equivalent linear programming problem.

		<i>B</i>		
		<i>B</i> ₁	<i>B</i> ₂	<i>B</i> ₃
<i>A</i>	<i>A</i> ₁	1	-1	3
	<i>A</i> ₂	3	5	-3
	<i>A</i> ₃	6	2	-2

6+4

2. (a) Given below is the relevant portion obtained after a few iterations of a linear programming problem under the simplex method, using the usual notations :

		<i>C</i> _j	5	3	0	0	M	M
<i>C</i> _B	Basic Variable (<i>B</i>)	Quantity <i>b</i> = <i>x</i> _B	<i>x</i> ₁	<i>x</i> ₂	<i>s</i> ₁	<i>s</i> ₂	<i>A</i> ₁	<i>A</i> ₂
3	<i>x</i> ₂	5/2	0	1	5/16	1/8	0	⋮
M	<i>A</i> ₁	3	0	0	-3/8	1/4	1	
5	<i>x</i> ₁	1	1	0	-1/8	-1/4	0	

Obtain the optimal solution of the above linear programming problem.

- (b) A firm has a single channel service station with the following arrival and service time probability distributions :

<u>Inter-arrival time</u> (minutes)	<u>Probability</u>	<u>Service time</u> (minutes)	<u>Probability</u>
10	0.10	5	0.08
15	0.25	10	0.14
20	0.30	15	0.18
25	0.25	20	0.24
30	0.10	25	0.22
		30	0.14

The queue discipline is FCFS. The process begins at 10 am. If the attendant's wages are ₹ 100 per hour and customer's waiting time costs ₹ 150 per hour, then would it be economical to engage a second attendant? You are given the following random numbers :

Arrival : 20, 73, 30, 99, 66, 83, 32, 75, 04, 15, 29, 62, 37, 68, 94.

Service : 26, 43, 98, 87, 58, 90, 84, 60, 08, 50, 37, 42, 28, 84, 65.

4+6

3. (a) A newsboy purchases newspaper everyday in bulk. The surplus at the end of the day are dead loss. A newspaper costs him ₹ 5 and it solds for ₹ 6 each. The daily demand of newspaper varies from 96 to 100.

Obtain the pay-off matrix and obtain decision by different principles of decisions making under uncertainty with index of optimism (α) = 0.7.

- (b) The following loss matrix is an initial basic feasible solution of a maximisation problem. Each cell value is obtained by subtracting each cell profit value from the highest profit of the matrix, which is ₹ 25. The loss per unit (in ₹) is given at the right-hand top corners of each cell and the encircled values are the allocation (in units) in each cell.

From \ To		Market				Availability
		M ₁	M ₂	M ₃	M ₄	
Warehouse	W ₁	13	7 200	19	0	200
	W ₂	17	18 ?	15	7 380	500
	W ₃	11 180	22	14 100	5 ?	300
Demand		180	?	100	400	1000

Find out the optional solution and the maximum profit.

6+4

4. (a) Two competitors are competing for the market share of the similar product. The pay-off matrix in terms of their advertising plan is given below. Suggest optimal strategies for two firms and the value of the game.

		B		
		No advertising	Medium advertising	Heavy advertising
A	No advertising	10	5	-2
	Medium advertising	13	12	13
	Heavy advertising	16	14	10

Please Turn Over

- (b) Maxo Ltd., a multinational IT company, has three expert Software Professionals. The company wants three application software programs to be developed. The head of Program Development estimates the computer time (in minutes) required by the experts for development of Application Software Programs as follows:

Software programs	Computer time (in minutes) required by Software Professionals		
	Mr. A	Mr. B	Ms. C
1	600	510	420
2	300	∞	660
3	660	720	780

Software 2 is required to be developed in Linux environment. Mr. B is unable to work in Linux environment. Assign the software professionals to the application software programs to ensure minimum usage of computer time.

- (c) Find the dual of the following primal LPP:

$$\text{Maximise } Z = 5x_1 + 10x_2$$

Subject to,

$$x_1 + 2x_2 \leq 60$$

$$3x_1 + 4x_2 \leq 100$$

$$x_1 \geq 0 \text{ and } x_2 \text{ is unrestricted in sign.}$$

4+3+3

Module - II

Answer *any two* questions.

5. (a) A factory has 1000 bulbs installed. The cost of individual replacement is USD 3/- while that of group replacement USD 1/- per bulb, respectively. It is decided to replace all the bulbs simultaneously at fixed intervals and also to replace the individual bulbs that fail in between. Determine optimal replacement policy. Failure probabilities are as given below : (No need for converting USD into INR, where USD stands for US Doller and INR means Indian Rupee)

Week	1	2	3	4	5
Failure Probability (P)	0.10	0.25	0.50	0.70	1.00

- (b) Customers arrive at a one man barber shop according to Poisson Process with a mean interarrival time of 20 minutes. Customers spend an average of 15 minutes in the barber chair. If an hour is used as a unit of time, then
- What is the probability that a customer need not to wait for a haircut?
 - What is the expected number of customers in the barber shop and in the queue?

- (iii) How much time can a customer be expected to spend in the barber shop?
- (iv) The owner of the shop will provide one another chair and hire another barber when a customer's average time in the shop exceeds 1.25 hrs. By how much should the average rate of arrivals increase in order to justify a second barber.
- (v) What is the probability that there will be 6 or more customers waiting for service? 5+5
6. (a) Eight jobs are to be processed at three machines A, B, C in the order CBA. The time taken by each job on the three machines is given below. Each machine can process one job at a time. All the processing times provided for these machines are in minutes.

Jobs	1	2	3	4	5	6	7	8
Machine A	7	8	6	6	7	8	5	7
Machine B	2	2	1	3	3	2	4	2
Machine C	6	5	4	4	2	1	5	5

Determine the optimal sequence for the jobs and calculate total Elapsed time and Idle time for all the machines.

- (b) A project consists of nine activities whose time estimates (in weeks) and other characteristics are given below :

Activity	Preceding Activity(ies)	Time estimates (weeks)		
		Most Optimistic	Most Likely	Most Pessimistic
A	-	2	4	6
B	-	6	6	6
C	-	6	12	24
D	A	2	5	8
E	A	11	14	23
F	B, D	8	10	12
G	B, D	3	6	9
H	C, F	9	15	27
I	E	4	10	16

- (i) Show the PERT network for the project.
- (ii) Identify the critical activities.
- (iii) What are the expected project completion time and its variance?
- (iv) What is the probability of completing the project two weeks before the expected time?

Please Turn Over

- (v) If the project is required to be completed by March 31 of 2024 and the manager is 95% confident to meet the deadline, when should he start the project work? 5+5

Given,	Z	0.31	0.61	1.645	2.33
Area		0.1217	0.2291	0.45	0.49

7. (a) Find the optimal order quantity for a product, the price breaks for which are as follows :

Order Quantity	Price per unit (₹)
$0 \leq Q_1 < 500$	100.00
$500 \leq Q_2 < 750$	92.50
$750 \leq Q_3$	87.50

The monthly demand of the product is 200 units, the holding cost is 2 percent of the unit cost and the ordering cost is ₹ 1000.

- (b) There are six jobs, each of which must go through the machines A and B in the order AB. Processing times are given below.

Machines/Jobs	1	2	3	4	5	6
Machine A(Hours)	30	120	50	20	90	110
Machine B(Hours)	80	100	90	60	30	10

Determine a sequence for the six jobs that will minimize the total elapsed time. Find also the total elapsed time and idle time for machine A and B. 5+(2+1+2)

8. (a) The demand for an item in a manufacturing company is 18000 units per year and the company produces the item at a rate of 3000 per month. The cost of one set up is ₹ 500 and the holding cost of one unit per month is ₹ 0.15. The shortage cost of one unit is ₹ 240 per year.

Determine the optimum manufacturing quantity and the optimum number of shortages. Also determine optimal inventory level and total variable cost.

- (b) An entrepreneur is considering purchasing a machine for his factory. The related data for alternative machines are as follows :

	Machine P	Machine Q	Machine R
Life (years)	10	10	10
Salvage Value (₹)	500	1,000	1,200
Present Investment (₹)	12,000	14,000	17,000
Total Annual Cost (₹)	3,000	2,000	1,500

As an advisor of the company, you have been asked to select the best machine considering a 12% normal rate of return per year. Given that :

Present worth factor series @ 12% for 10 years is 5.6502

Present worth factor @ 12% for 10th year is 0.3220.

(2+1+1+1)+5