

2022

## ADVANCED BUSINESS MATHEMATICS — HONOURS

Paper : DSE-5.1AH

(Module - II)

Full Marks : 40

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **any four** questions.

1. (a) Evaluate :  $\log_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{x}$

(b) The function  $f(x) = \frac{2x^2 - 8}{x - 2}$  is undefined at  $x = 2$ . What value must be assigned to  $f(2)$ , if  $f(x)$  is to be continuous at  $x = 2$ ? 5+5

2. (a) Find the matrices  $A$  and  $B$  for which  $2A + B = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$  and  $3B - 2A = \begin{bmatrix} 10 & 1 \\ 3 & 5 \end{bmatrix}$ .

(b) Prove that :  $\begin{vmatrix} x^2 + y^2 + 1 & x^2 + 2y^2 + 3 & x^2 + 3y^2 + 4 \\ y^2 + 2 & 2y^2 + 6 & 3y^2 + 8 \\ y^2 + 1 & 2y^2 + 3 & 3y^2 + 4 \end{vmatrix} = x^2 y^2$ . 5+5

3. (a) Evaluate :  $\int \frac{(x+2)}{\sqrt{x-2}} dx$

(b) Evaluate :  $\int_2^5 \frac{dx}{(x+3)(x+5)}$  5+5

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4. (a) Evaluate :  $\int \frac{x dx}{\sqrt{2x+3} + \sqrt{5+2x}}$

(b) If  $y = ae^{Mx} + be^{-Mx}$ , show that  $\frac{d^2y}{dx^2} - M^2y = 0$ . 5+5

5. (a) The demand function of a firm is  $3p + x = 48$ , where  $p$  is the price per unit and  $x$  is the number of units demanded. Find the level of output where total revenue is maximised.

(b) Find the area bounded by the straight lines  $3x + 4y = 12$ ,  $x = 1$  and  $x$ -axis. 5+5

6. (a) If  $f(x) = \frac{1-x}{1+x}$ , find  $f\left\{f\left(\frac{1}{x}\right)\right\}$ , ( $x \neq 0$ ).

(b) Find  $\frac{dy}{dx}$  when  $y = x^x + x^2$ . 5+5

7. (a) Solve by Cramer's Rule :  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ ,  $\frac{2}{x} + \frac{5}{y} + \frac{3}{z} = 0$ ,  $\frac{1}{x} + \frac{2}{y} + \frac{4}{z} = 3$ .

(b) If  $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$ , show that  $AB = 6I_3$  ( $I_3$  is the identity matrix of order 3).

Utilise this result to solve  $2x + y + z = 5$ ,  $x - y = 0$  and  $2x + y - z = 1$ . 5+5

8. (a) Solve for  $x, y, z$  and  $t$  if  $\begin{bmatrix} x-z & -x-z \\ 7-t & 6+z \end{bmatrix} = \begin{bmatrix} 3-t & 5-t \\ t+5 & x-y \end{bmatrix}$ .

(b) Find the value of  $x$ , when  $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x+1 & -1 \\ -1 & 1 & x+1 \end{vmatrix} = 0$ . 5+5