

2023

## ADVANCED BUSINESS MATHEMATICS — HONOURS

Paper : DSE-5.1 AH

(Module - II)

Full Marks : 40

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words  
as far as practicable.*Answer **any four** questions.

1. (a) Evaluate :  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$ .

(b) The function  $f(x) = \frac{x^3 - 8}{x^2 - 4}$  is undefined at  $x = 2$ . Redefine the function so as to make it continuous at  $x = 2$ . 5+5

2. (a) Evaluate :  $\int_{-1}^0 \frac{1+x}{1-x} dx$ .

(b) Evaluate :  $\int \frac{dx}{(x-3)\sqrt{x+1}}$ . 5+5

3. (a) If  $f(x) + 2f(-x) = x^2$ , show that  $f(3) = 3$ .

(b) If  $y = 2x^3 + 3x^2 - 36x + 7$ , find the values of  $x$  for which  $\frac{dy}{dx} = 0$ . 5+5

4. (a) Solve by Cramer's Rule :  $2x - y + 3z = -2$ ,  $3x + 2y + 3z = 9$ ,  $x + 2y - z = 9$ .

(b) Show that the matrix  $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$  is orthogonal. Hence, find  $A^{-1}$ . 5+5

Please Turn Over

5. (a) Evaluate :  $\int (2x+1)\sqrt{4x^2+4x+5} dx$

(b) If  $x = t^2 + 2t$ ,  $y = t^2 - 2t$ , find out  $\frac{d^2y}{dx^2}$  at  $t = 1$ .

5+5

6. (a) Without formal expansion prove that  $\begin{vmatrix} x & y & 0 \\ 0 & x & y \\ y & 0 & x \end{vmatrix} = x^3 + y^3$ .

(b) Prove that  $\left(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$  is a factor of  $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ .

5+5

7. (a) The price per unit  $p(x)$  at which a company can sell all that it produces is given by  $p(x) = 300 - 4x$  and that cost function is  $c(x) = 300 + 52x$ , where  $x$  is the number of units produced. Find  $x$  so that profit is maximum.

(b) Find the area bounded by straight lines  $3x - 2y = 6$ ,  $2x + y = 10$  and  $x$  axis.

5+5

8. (a) Let  $A$  and  $B$  be two matrices such that  $AB = \begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix}$  and  $B = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ . Find out  $A$ .

(b) If  $P = \begin{pmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{pmatrix}$ , then show that  $P^2 = P$  and hence find matrix  $Q$  such that  $4P^2 - 2P + Q = O$ ,

where  $O$  is the zero matrix of order 3.

5+5