# 2023

## COMPUTER SCIENCE — HONOURS

**Paper : CC-6**

**(Computational Mathematics)**

**Full Marks : 50**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***question no.* 1** and ***any four*** questions from the rest.

**1.** Answer ***any five*** questions : 2×5

(a) Prove by mathematical induction that the proposition P : the sum of the first $n$ odd numbers is $n^2$.

(b) If $y = 4x^6 - 5x$, find the percentage error in $y$ at $x = 1$, if the error in $x = 0.04$.

(c) When is a function said to be an invertible function?

(d) What is the condition for convergence of Gauss-Jacobi iteration method in solving a set of linear equations?

(e) Briefly state the Generalized Pigeonhole Principle.

(f) When is a graph said to be a regular graph? Draw a regular graph of degree 4 having 4 vertices.

(g) Find the coefficient of $x^5y^8$ in the expansion of $(x + y)^{13}$ by using Binomial theorem.

(h) Find the first five terms of the sequence defined by the following recurrence relation and initial conditions :

$$a_n = a_{n-1} + 3a_{n-2},\ a_0 = 1,\ a_1 = 2$$

**2.** (a) Show that $x^4 + 9x^3 + 4x + 7$ is $O(x^4)$.

(b) Define power set of a set S. What is the power set of the empty set? What is the power set of $\{\phi\}$?

(c) Justify the statement — "Among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4". 4+3+3

**3.** (a) How many bit strings of length 8 either starts with a 1 or ends with 10?

(b) If $X$ is a random variable over a sample space S, E is the expected value and $\mu$ is the mean, prove that $\text{var}(X) = E(X^2) - \mu^2$.

(c) A person dealt 5 cards from an ordinary 52-card deck. Find the probability $p$ that they are all spades. 4+4+2

4. (a) Prove that the number of $m$-combinations of a set with $n$ elements, where $n$ is a non-negative integer and $m$ is an integer $0 \le m \le n$ is

$$C(m,n) = \frac{n!}{m!(n-m)!}$$

(b) Give the definition of a linear homogeneous recurrence relation of degree $m$ and having constant coefficients.

(c) Solve the recurrence relation $a_n = 6a_{n-2} - 5a_{n-1}$ with initial conditions $a_0 = 2$ and $a_1 = 10$. 4+2+4

5. (a) A committee of 5 Principals is to form from a group of 6 male Principals and 8 female Principals. If the selection is made randomly, find the probability that there are 3 female Principals and 2 male Principals.

(b) State the Bayes' Theorem on conditional probability.

(c) Use Newton's backward interpolation technique to compute $f(22)$ for the following table :

| $x$ | 0 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| $f(x)$ | 2.5 | 3.0 | 4.2 | 7.6 | 18.7 |

Write the formula used. 4+2+4

6. (a) Write an algorithm to find the roots of an equation using bisection method.

(b) Prove that in a simple connected graph with $n$ vertices ($n > 1$), at least two vertices are of equal degrees.

(c) What is a minimum spanning tree? 5+3+2

7. (a) Prove that the sum of the degrees of the vertices of a graph $G$ is equal to twice the number of edges in $G$.

(b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y = 1$ at $x = 0$. Find $y$ for $x = 0.1$ by using Euler's method and taking step size = 0.02. Write down the formula used. 5+5

8. (a) Evaluate $\int_0^6 \frac{dx}{(1+x)^2}$, taking six equal intervals, correct to 3-decimal places. Show the formula used.

(b) Write the algorithm for linear regression. Write down the formula. 5+5