

2024

PHYSICS — HONOURS

Paper : CC-11

(Electromagnetic Theory)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer *question no. 1* and *any four* questions from the rest.

1. Answer *any five* questions : 2×5
- (a) The average light intensity on the earth's surface is $1.3 \times 10^3 \text{ J/m}^2 \cdot \text{s}$. What is the rms value of the electric field assuming the light is monochromatic?
- (b) A plane electromagnetic wave is incident normally at the boundary of two dielectrics of refractive indices n_1 and n_2 ($n_1 < n_2$). If the transmission coefficient is 0.80, find the value of $\frac{n_2}{n_1}$.
- (c) Show that the angle between the reflected ray and the transmitted ray is 90° when the ray is incident at Brewster's angle on the surface of a dielectric.
- (d) In a medium where the conduction current density is given by
- $$\vec{J} = [3 \sin(\omega t - 10z) \hat{j} + \cos(\omega t - 10z) \hat{k}] \text{ A/m}^2,$$
- find the volume charge density, that depends on time.
- (e) What is skin depth of a conductor?
- (f) A sugar solution in a tube 20 cm long produces an optical rotation of 13° with light of wavelength 600 nm. Find the strength of the solution. Given, specific rotation of sugar is 65° .
- (g) How can a quarter wave plate be used to produce circularly polarized light?
2. (a) Show that the displacement current in a parallel plate capacitor is equal to the conduction current in the connecting leads.
- (b) An electromagnetic wave is propagating from one linear dielectric medium to another with no free charge and current. Write the boundary conditions for the electric and magnetic fields.
- (c) For a plane electromagnetic wave propagating in a source-free region, show that Maxwell's equations reduce to $\vec{k} \times \vec{E} = \omega \vec{B}$, $\vec{k} \cdot \vec{B} = 0 = \vec{k} \cdot \vec{E}$, where \vec{k} is the wave vector, \vec{E} and \vec{B} are electric and magnetic fields and ω is the frequency of the wave. 3+4+3

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3. (a) Write Maxwell's equations in free space. Obtain the wave equation for electric field from them.
 (b) Find the magnitude and direction of the Poynting vector at the surface of a long straight wire of circular cross section carrying a direct current I . The radius of the wire is ' a ' and resistance per unit length is ' R '.
 (c) Show that the momentum density in electromagnetic field is given by \vec{S}/c^2 in vacuum, where \vec{S} is the Poynting vector. 3+3+4
4. (a) Write Maxwell's equations for a linear conducting medium. Show that any initial free volume charge dissipates in a characteristic time.
 (b) The wave number of an electromagnetic wave propagating through a conductor can be written as $k = k_R + ik_I$ when,

$$k_R = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1 \right]^{\frac{1}{2}}$$

$$\text{and } k_I = \omega \sqrt{\frac{\mu\epsilon}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1 \right]^{\frac{1}{2}}.$$

Show that the skin depth

(i) of a good conductor is $\frac{\lambda}{2\pi}$.

(ii) of a poor conductor is $\frac{2}{\sigma} \sqrt{\frac{\epsilon}{\mu}}$.

Symbols have their usual meaning.

- (c) The conductivity of a metal is $5.8 \times 10^7 (\Omega\text{m})^{-1}$. Find the attenuation of an electromagnetic wave of frequency 1 kHz by a plate of thickness 0.1 m made of that metal. (1+2)+4+3
5. (a) Using Fresnel's theory of optical activity, show that the angle of rotation of the plane of vibration

$$\delta = \frac{\phi}{2} = \frac{\pi}{\lambda} d (\mu_L - \mu_R),$$

where ϕ = phase difference, μ_L and μ_R = refractive indices of the crystal w.r.t. left-handed and right-handed vibrations respectively, d = thickness of the crystal, λ = wavelength of light.

- (b) What is Babinet Compensator? Show that total path difference between E and O-rays for two crystals is $\Delta = (d_1 - d_2)(\mu_E - \mu_O)$, where d_1, d_2 are the widths of the 1st and 2nd crystals respectively, and μ_E, μ_O are the refractive indices of the E and O-rays.
 (c) Critical angle for refraction for a certain material to air is 45° . Calculate the Brewster's angle for it. 3+(2+3)+2

6. (a) What is a quarter wave plate? Discuss with diagram how it can be used to produce elliptically polarized light.
- (b) Find the state of polarization when the x and y components of the electric field are
- (i) $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz)$
- (ii) $E_x = E_0 \cos(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz + \pi)$.
- (c) A plane polarized light of wavelength 550 nm changes to circularly polarized light on passing through a quartz crystal cut parallel to optic axis. Calculate the minimum thickness required to produce such effect.
- [Given that $n_e - n_o = 0.005$ where n_e and n_o are the refractive indices of E-ray and O-ray respectively].
- 4+3+3
7. (a) Define (i) optic axis, (ii) principal section of a crystal and (iii) principal plane.
- (b) Two Nicol prisms are so arranged that the amount of light transmitted through them is maximum. What are the percentage reduction of intensity of light when the analyzer is rotated through
- (i) 30° and (ii) 90° ?
- (c) State the working principle of biquartz plate. (2+2+2)+2+2