

2024

PHYSICS — HONOURS

Paper : CC-12

(Statistical Physics)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any four** questions from the rest.

1. Answer **any five** questions : 2×5
- (a) Assuming the entropy S is a function of number of microstates Ω , show that the additive character of S and the multiplicative character of Ω necessarily require $S \propto \ln \Omega$.
 - (b) Show that for a canonical system, the mean energy $\langle E \rangle = -\frac{\partial}{\partial \beta} (\ln Z)$, where Z is the canonical partition function.
 - (c) State the postulate of equal a priori probabilities.
 - (d) Draw the nature of Fermi-Dirac distribution function at absolute zero and for two other temperatures T_1, T_2 where $T_2 > T_1$.
 - (e) In how many ways can five Bosons be arranged in three quantum states?
 - (f) What is meant by 'ultraviolet catastrophe'?
 - (g) What is λ -transition in liquid Helium?
 - (h) How the pressure of a non-relativistic free Fermi gas in three dimension depends, at $T = 0K$, on the density of fermions ' n '?
2. (a) A system of N non-interacting classical particles, each of mass m is in a two-dimensional harmonic potential of the form $V(r) = \alpha(x^2 + y^2)$, where α is a positive constant. Find the canonical partition function of the system at temperature T .
- (b) Calculate Helmholtz free energy of the system mentioned in (a).
- (c) A system of harmonic oscillators has two normal modes of vibration, with frequencies ω_1 and $\omega_2 = 2\omega_1$. What is the probability that at temperature T , the system has an energy less than $4 \hbar \omega_1$? 4+2+4

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3. (a) Show that for a canonical system $\langle (E - \bar{E})^2 \rangle = k_B T^2 C_V$, where C_V is the heat capacity at constant volume.
- (b) Write the Bose-Einstein distribution formula. Starting from BE distribution formula, deduce Planck's law of black-body radiation.
- (c) Show that Planck's law of radiation reduces to Wien's distribution law for $h\nu \gg k_B T$ and to Rayleigh-Jeans law for $h\nu \ll k_B T$. 2+(1+3)+(2+2)

4. (a) Show that the mean energy per electron at $T = 0\text{K}$ is $\frac{3}{5} E_F$, where E_F denotes the Fermi energy for the electrons in a metal.
- (b) Find the Fermi temperature of the electron gas in typical metal.

Given :

the number density of electrons in the metal $= 6.0 \times 10^{28} \text{ m}^{-3}$;

the Boltzmann constant $k_B = 1.38 \times 10^{-23} \text{ J/K}$;

the electronic mass $m = 9.1 \times 10^{-31} \text{ kg}$.

Hence show that the electron gas in a metal is degenerate at ordinary temperature.

- (c) Show that the range (Δf) of the single particle occupation number for a fermion at temperature T having an energy within $\pm k_B T$ of the Fermi energy is approximately 0.46. Here $\Delta f = f(E_F - k_B T) - f(E_F + k_B T)$. 4+(3+1)+2
5. (a) For a fermion gas in two dimension with a linear energy-momentum relation $E = |\vec{p}| C$, where $\vec{p} = (\hat{i}p_x + \hat{j}p_y)$ and C is a constant. If σ is the number of electrons per unit area, find energy per unit area of the fermion gas.
- (b) The specific heat of silver at low temperatures can be represented by $C_V = \gamma T + \alpha T^3$, where γ and α are constants. Explain the origin of the term linear in T .
- (c) In terms of number density (n) and absolute temperature (T), find the condition to be satisfied by an ideal gas for the onset of quantum effects. 4+3+3

6. (a) Consider two indistinguishable particles (1 and 2) which may exist in two different states (a and b). The function for the entire system will be one of the following two possible forms :

$$\psi_1 = \frac{1}{\sqrt{2}} [\psi_a(1)\psi_b(2) + \psi_a(2)\psi_b(1)] \quad \text{and} \quad \psi_2 = \frac{1}{\sqrt{2}} [\psi_a(1)\psi_b(2) - \psi_a(2)\psi_b(1)]$$

Identify with reason the wave function that will represent the system when (i) both are Bosons, (ii) both are Fermions.

(3)

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- (b) A system can have three energy levels : $E = 0, \pm \epsilon$. The level $E = 0$ is doubly degenerate, while the others are non-degenerate. Write down the partition function and find the average energy of the system.
- (c) The chemical potential of photon gas is Zero. — What is the physical significance of this statement?
4+4+2
7. (a) Distinguish between microcanonical, canonical and grand canonical ensembles.
- (b) Gas molecules of mass m are confined in a cylinder of radius R and height L (with $R \gg L$) kept vertically in the Earth's gravitational field. Find the average energy of the gas at low temperatures such that $mgL \gg k_B T$.
- (c) What is a Bose-Einstein condensate? Is it possible to get a similar condensate for a system of non-interacting electrons?
3+4+3
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