2025

MATHEMATICS — GENERAL

Paper : SEC-B-2

(Boolean Algebra)

Full Marks : 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Marks : 20)

1. Choose the correct option and justify your answer : (1+1)×10

(a) Let $S = \{1, 2, 3\}$ and consider the relation $R = \{(2, 2)\}$ on S. Then R is

(i) symmetric but not antisymmetric (ii) antisymmetric but not symmetric

(iii) symmetric as well as antisymmetric (iv) neither symmetric nor antisymmetric.

(b) Let $(S, \leq)$ be a poset and $P = \{a, b\}$ be a subset of S. Then it is true that P has always

(i) more than one upper bounds (ii) only one upper bound

(iii) more than one l.u.b. (iv) unique l.u.b. if it exists.

(c) The complement of $(x' + y)(x' + y')$ is

(i) $(x + y)(x + y')$ (ii) x

(iii) $(x' + y')\, y'$ (iv) $(x + y)(x' + y')$.

(d) It is false that from a Hasse diagram of some poset

(i) minimal element(s) can be determined

(ii) maximal element(s) can be determined

(iii) both maximum and minimum element(s) can be determined

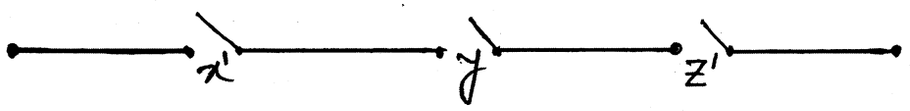
(iv) maximum and minimum element(s) can never be determined.

(e) Let $(B, \wedge, \vee)$ be a lattice and $a, b, c \in B$. Then dual of $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$ is

(i) $a \wedge (b \vee c) = (a \wedge b) \wedge (a \wedge c)$ (ii) $a \wedge (b \wedge c) = (a \wedge b) \vee (a \wedge c)$

(iii) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ (iv) $a \vee (b \vee c) = (a \wedge b) \vee (a \vee c)$.

(f)



Boolean expression corresponding to the above circuit is written as

(i) $x'yz'$ (ii) $xy'z$

(iii) $x' + y + z'$ (iv) $x + y' + z$.

(g) In the Boolean algebra which one of the following is not true?

(i) $a + a = a$ (ii) $a + 1 = 0$

(iii) $a \cdot (a + b) = a$ (iv) $(a + b)' = a' \cdot b'$.

(h) Let X be a non-empty set and $(P(X), \subseteq)$ be a poset of all subsets of X. If $A, B \in P(X)$ then $\inf(A, B)$ is

(i) A (ii) B

(iii) $A \cup B$ (iv) $A \cap B$.

(i) The number of reflexive relations defined on a set S containing n elements is

(i) 2^{n^2} (ii) 2^{n^2-n}

(iii) $2^{\frac{n(n-1)}{2}}$ (iv) $2^{\frac{n(n+1)}{2}}$.

(j) The complete CNF of the function x in two variables x, y is

(i) $(x + y)(x' + y)(x + y')(x' + y')$ (ii) $(x + y)(x' + y)(x + y')$

(iii) $(x + y)(x' + y')$ (iv) none of these.

Group - B

(Marks : 60)

Answer ***any six*** questions.

2. (a) Define greatest and least element in a poset.

(b) Draw the Hasse diagram of the poset $(S, \leq)$, where $S = \{4, 12, 24, 48, 72\}$ and $a \leq b$ means a divides b for all $a, b \in S$. Find the greatest and least element (if exists) in $(S, \leq)$.

(c) Let $S = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ and consider the poset $(S, \leq)$ where $a \leq b$ iff a divides b for all $a, b \in S$. Find the upper bound(s) and least upper bound of the set $\{6, 15\}$ in $(S, \leq)$. 2+(2+2)+(2+2)

3. (a) Define minimal and maximal element in a poset.

(b) Verify the statement : In a poset there exists an element which is both minimal and maximal.

(c) Let $S = \{1, 2, 3\}$ and T be the set of all proper non-empty subsets of S. Find the minimal elements and maximal elements in the poset $(T, \leq)$ where $\leq$ is the set inclusion relation. 2+2+(3+3)

4. (a) Define distributive lattice.

(b) Let S be the set of all positive divisors of 20 and a partial order relation $\leq$ is defined on S by $a \leq b$ iff a divides b. Examine whether $(S, \leq)$ is a distributive lattice.

(c) Let $(L, \wedge, \vee)$ be a distributive lattice and $a, b, c, \in L$.

Prove that $a \wedge c = b \wedge c$ and $a \vee c = b \vee c \Rightarrow b = a$. 2+3+5

5. (a) Let $(S, \leq)$ be a poset and $a, b \in S$. Prove that $a \vee b = b$ iff $a \wedge b = a$.

(b) Let $(L, \leq)$ be a lattice and $a, b \in L$. Show that $a \wedge (a \vee b) = a$.

(c) Give one example of a poset where there are more than one minimal elements but no smallest element. 4+3+3

6. (a) Let $(B, +, \cdot, ')$ be a Boolean algebra and a, b, c $\in$ B.

Prove that $a{\cdot}c + a'{\cdot}b + b{\cdot}c + (a + b){\cdot}(a' + c)$

(b) Let $(B, \vee, \wedge, ')$ be a Boolean algebra and $a \in B$. Prove that $a \vee a = a$ and $(a')' = a$. 4+(3+3)

7. (a) Show that the poset of the divisors of 60 ordered by divisibility is a lattice and interpret their meet and join.

(b) Define complete lattice. Prove that dual of a complete lattice is complete.

(c) Define homomorphism between two lattices. (2+1+1)+(1+3)+2

8. (a) Use distributive law of Boolean Algebra B to show that

$ab + cd = (a + c)\ (b + c)\ (a + d)\ (b + d)$ for all $a, b, c, d \in B$.

(b) Transform the following CNF into an expression in DNF :

$(x + y' + z)\ (x + y + z')\ (x + y' + z')\ (x' + y + z)\ (x' + y + z')\ (x' + y' + z)$

(c) Show that in a Boolean Algebra B, for all $a, b \in B$,

(i) $(a' + ab)' = ab'$

(ii) $[\,(a' + b)' \cdot (a + b')\,]' = a' + b$. 2+4+(2+2)

9. (a) Construct the truth table for the Boolean expression :

$xy' + y\ (x' + z)$

(b) Simplify the Boolean polynomial

$(x_1 + x_2)\ (x_1 + x_3) + x_1\ x_2\ x_3$ by Karnaugh diagram. 5+5

Please Turn Over

10. (a) We wish a light in a room to be controlled independently by three wall switches at the three entrances of the room in such a way that flicking any one of them will change the state of the light (on to off and off to on). Design a simple series parallel switching circuit which will do the required job.

(b) Find a switching circuit which realizes the swithcing function f given by the following switching table :

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	0

5+5