

(d) If the Lagrangian does not depend upon time explicitly, then

$$(i) \sum_{j=1}^n \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} - L \text{ is conserved} \quad (ii) \sum_{j=1}^n q_j \frac{\partial L}{\partial q_j} - L \text{ is conserved}$$

$$(iii) \sum_{j=1}^n \dot{q}_j \frac{\partial H}{\partial \dot{q}_j} + H \text{ is conserved} \quad (iv) \sum_j p_j \frac{\partial H}{\partial p_j} - H \text{ is conserved.}$$

(e) The dynamics of a particle governed by the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx \dot{x}t$ describes

- (i) an undamped simple harmonic oscillator.
- (ii) a damped harmonic oscillator with a time varying damping factor.
- (iii) an undamped harmonic oscillator with a time dependent frequency.
- (iv) a free particle.

(f) The isotropy of space leads to the law of conservation of

- (i) linear momentum
- (ii) angular momentum
- (iii) energy
- (iv) parity.

(g) The Lagrangian for a simple pendulum is given by

$$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos\theta)$$

The Poisson bracket between θ and $\dot{\theta}$ is

$$(i) 1 \quad (ii) \frac{1}{ml^2}$$

$$(iii) \frac{1}{m} \quad (iv) \frac{g}{l}$$

(h) Given that the linear transformation of a generalized coordinate q and the corresponding momenta p ,

$$Q = q + 4\alpha p, \quad P = q + 2p$$

is canonical. The value of the constant α is

$$(i) 1 \quad (ii) \frac{1}{2}$$

$$(iii) \frac{1}{3} \quad (iv) \frac{1}{4}$$

(3)

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- (i) Let A, B and C be function of phase space variables. If $\{, \}$ represents the Poisson bracket, the value of $\{A, \{B, C\}\} - \{\{A, B\}, C\}$ is given by
- (i) 0 (ii) $\{\{C, A\}, B\}$
(iii) $\{B, \{C, A\}\}$ (iv) $\{A, \{C, B\}\}$.
- (j) The transformation of Lagrangian to Hamiltonian is done by
- (i) Laplace transformation (ii) Legendre transformation
(iii) Lorentz transformation (iv) Galilean transformation.

Group - B

Unit - 1

(Marks : 10)

2. Answer *any two* questions :

- (a) A bead of mass m is constrained to move on a straight frictionless wire AB , fixed at a point A on a vertical axis OA and rotating about OA with constant angular velocity ω . Set-up the Lagrangian and find the Lagrange's equation of motion. 4+1
- (b) State D'Alembert's principle. Derive Lagrange's equations of motion from it for conservative systems. 1+4
- (c) For a particle moving under a central force, construct Lagrange's equations of motion. 5
- (d) The Lagrangian for a system with 2 degrees of freedom is given by $L = \dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_1\dot{q}_2 - q_1^2 - q_2^2$. Use Lagrange's equation to find q_1 and q_2 . 5

Unit - 2

(Marks : 15)

3. Answer *any three* questions :

- (a) State the Hamilton's principle and derive the Hamilton's canonical equations of motion from it. 1+4
- (b) Consider a planet of mass m orbiting around the sun under the inverse square law of attraction $\frac{\mu m}{r^2}$, $\mu > 0$, then find the Hamiltonian of the system and also write down the Hamilton's equations of motion. 5

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- (c) (i) For two-dimensional harmonic oscillator, Lagrangian is given by

$$L = \frac{1}{2}m(\dot{r}^2 + \dot{r}^2\dot{\theta}^2) - \frac{1}{2}kr^2$$

Find the Hamilton's equations of motion.

- (ii) If the Lagrangian of a dynamical system is $L = \dot{q}_1^2 + \dot{q}_2^2 + k_1q_1^2$, then find the Hamiltonian of the system. 2+3
- (d) A particle of mass m is constrained to move on the surface of a cylinder defined by $x^2 + y^2 = R^2$. It is subjected to a force $F = -Kr$, proportional to its distance from the origin and directed to it. Using the Hamiltonian, write down the canonical equation of motion and show that motion in z -direction is simple harmonic. 4+1
- (e) For the Hamiltonian $H = p_1q_1 - p_2q_2 - aq_1^2 + bq_2^2$, solve the Hamilton's equation of motion and prove that $(p_2 - bq_2)/q_1 = \text{constant}$ and $q_1q_2 = \text{constant}$, where a, b are constants and q_1, q_2 are generalized co-ordinates, p_1, p_2 are corresponding generalized momentum. 3+1+1

Unit - 3

(Marks : 10)

4. Answer *any two* questions :

- (a) State and prove Liouville's theorem. 5
- (b) The Hamiltonian for a system described by the generalized co-ordinate x and generalized momentum p is

$$H = \alpha x^2 p + \frac{p^2}{2(1+2\beta x)} + \frac{1}{2}\omega^2 x^2$$

where α, β, ω are constants. Find the corresponding Lagrangian of the system. 5

- (c) Derive the Hamilton's equations of motion in spherical polar coordinates. 5
- (d) Lagrangian of an anharmonic oscillator of unit mass is $L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta \cdot x \cdot \dot{x}^2$, α and β being constants. Find the Hamiltonian and deduce the canonical equations of motion. 5

Unit - 4

(Marks : 10)

5. Answer *any two* questions :

- (a) (i) Show that if a function F does not depend on time explicitly and is a constant of motion, then its Poisson bracket with the Hamiltonian vanishes.
- (ii) Write the equations of motion in Poisson bracket form. 3+2

(5)

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(b) (i) Show that the transformation $q = P^2 + Q^2$, $p = \frac{1}{2} \tan^{-1} \left(\frac{P}{Q} \right)$ is canonical.

(ii) Find the canonical transformations defined by generating function

$$F_1(q, Q) = qQ - \frac{1}{2} m\omega q^2 - Q^2 / 4m\omega. \quad 2+3$$

(c) (i) If $[\varphi, \psi]$ be the Poisson bracket of φ and ψ , then prove that $\frac{\partial}{\partial t} [\varphi, \psi] = \left[\frac{\partial \varphi}{\partial t}, \psi \right] + \left[\varphi, \frac{\partial \psi}{\partial t} \right]$.

(ii) The Poisson bracket of q and $f(p)$ is given by $[q, f(p)] = f(p)$. Find $f(p)$. 3+2

(d) What do you understand by Hamilton's principal function? Use it to solve the problem of a particle falling freely under the action of gravity. 5