2025

PHYSICS — HONOURS

Paper : DSE-A-2.2

(Advanced Classical Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) Write the Lagrangian of a particle of mass 'm' moving in a central potential and show that its angular momentum is conserved.

(b) Show that the Poisson bracket of $[q_j, q_k] = 0$, where q represents generalized coordinate.

(c) Set up the Hamiltonian of a system for which the Lagrangian is given by

$$L(x,\dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^2 + \beta x\dot{x}^2 \ ,$$

where α and β are constants.

(d) Consider a particle moving in one-dimension under the action of a potential $V(x) = x^3 - x$. Find the stable equilibrium point of the particle, if any.

(e) Consider a thin uniform rectangular sheet of mass M extending from $x = 0$ to $x = a$ and $y = 0$ to $y = b$. Calculate I_{zz}, where the z axis is to be chosen to pass through the centre of the lamina.

(f) Using Virial theorem in classical mechanics, show that the average kinetic energy of a system, moving under restoring force, is equal to its average potential energy.

(g) What do you mean by dissipative dynamical system? Explain with one example.

Group - B

2. Answer ***any three*** questions :

(a) Using plane polar (r, θ) coordinates, find the geodesics on a two-dimensional flat plane. 5

(b) Write down the Lagrangian for a spherical pendulum where a point mass is suspended by a light inextensible string and moves under the action of gravity. Find the momenta. Find the Hamiltonian. 1+2+2

(c) Two point masses m_1 and m_2 are at rest on a smooth horizontal surface. The masses are connected by a light elastic spring of spring constant k of equilibrium length l_0. Assuming the motion to be in one dimenstion, write the T and V matrices for small amplitude oscillations of the coupled system. Find the normal frequencies and interpret their significance. 2+2+1

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(d) For a thin uniform square plate of side a and mass m, derive principal moment of inertia. Also calculate the moment of inertia about a diagonal. 3+2

(e) Find the fixed points for the map $x_{n+1} = x_n^2$ and determine their stability. 5

Group - C

Answer ***any four*** questions.

3. (a) In brachistochrone problem, show that for a particle of mass m moving under uniform gravitational field with zero initial veclocity, the path of minimum time is cycloid.

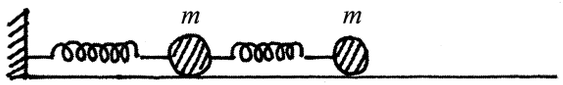
(b) Consider a function $z = \overline{f(x,y)}$. Use the Legendre transformation to form $w = g(u, v)$, where

$$u = \frac{\partial f}{\partial x} \text{ and } v = \frac{\partial f}{\partial y}.$$ 6+4

4. (a) A particle of mass m moves with uniform velocity $\vec{v}$ in a rotating frame whose angular velocity ($\vec{\omega}$) is constant. Write down the Lagrangian of the particle and corresponding velocity dependent potential. Solve Lagrange equation of motion and identity Corioli's force and centrifugal force.

(b) A flexible chain is suspended between two fixed points. The length of the chain is fixed. Find the curve that will minimize the gravitational potential energy of the chain. (2+3)+5

5. (a) Two identical particles of mass m lying on a horizontal table are connected to identical massless spring of spring constant k, as shown in figure.



Show that the characteristic frequencies of normal mode of oscillation are given by

$$\left[\left(\frac{3 \pm \sqrt{5}}{2}\right)\frac{k}{m}\right]^{1/2}.$$

(b) Show that Poisson bracket $[L_x, L_y] = L_z$, where L_x, L_y and L_z are the cartesian components of angular momentum vector. Also show that

$$[L_x,[L_y,L_z]]+[L_y,[L_z,L_x]]+[L_z,[L_x,L_y]]=0.$$ 5+(2+3)

6. (a) Find the conditions on the real parameters α, β, γ and δ, such that

$$\dot{q} = \alpha q + \beta p$$
$$\dot{p} = \gamma q + \delta p$$

are Hamilton's equation of motion for some $H(p, q)$. Hence obtain $\dot{H}(p,q)$.

(b) For a homogeneous cube of mass m and side a
(i) set up the principal axes with origin at the centre of mass of the cube.
(ii) Calculate the principal moment of inertia I_1, I_2 and I_3. (3+2)+(2+3)

7. (a) What do you mean by a 'dynamical system'? Cast the equation of damped harmonic oscillator, $\ddot{x}+2b\dot{x}+\omega^2 x=0$, into a dynamical system. What is its dimensionality?

(b) Find the fixed points of the dynamical system represented by $\dot{x}=x^2-1$. Draw the phase portrait and the flow of x. Hence ascertain the nature of the fixed points. (2+1+1)+(2+2+2)

8. (a) Consider a van der pol oscillator $\ddot{x}-\mu\left(x^2-1\right)\dot{x}+\beta x=0$ with $\mu>0$. State the conditions for which the system would be conservative, dissipative and anti-dissipative.

(b) Consider the logistic map $x_{n+1}=rx_n(1-x_n)$ for $0\le x\le 1$ and $0\le r\le 4$. Find all the fixed points.

(c) Assume the predator-prey model given by $\dot{x}=x(a-by)$ and $\dot{y}=y\left(cx-d\right)$, where a, b, c, d are all positive constants, find out the fixed points for the system. Using Jacobian matrix, identify the nature of the fixed points. 3+3+(1+3)
