2025

PHYSICS — HONOURS

Paper : DSCC-5

(Modern Physics)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions from the rest.

1. Answer ***any five*** questions : 3×5

(a) What is group velocity? Show that the group velocity of the de Broglie wave is equal to the velocity of the particle.

(b) Plot the energy density of a perfect black body as a function of wavelength for two different temperatures T_1 and T_2 $(T_1 > T_2)$.

(c) Show that the momentum operator is Hermitian for the class of functions which go to zero at infinity.

(d) A particle of mass m has the wave function

$$\Psi(x,t) = Ae^{-a\left[\left(\frac{mx^2}{\hbar}\right)+it\right]},$$

where A and a are positive real constants. Find A using normalization.

(e) The wave function of a particle at time $t=0$ is given by $\psi(x,0) = \frac{2}{a^{3/2}}xe^{-x/a}$ for $x > 0$ (a = constant). Find its momentum space wave function at time $t = 0$.

(f) Find the constant B which makes $e^{-\alpha x^2}$ (α = constant) an eigenfunction of the operator $\left(\frac{d^2}{dx^2} - Bx^2\right)$.

(g) Show that if ψ be an eigenfunction of the operator $\hat{A}$ with eigenvalue λ, then it is also eigenfunction of e^{A} with eigenvalue e^{λ}.

(h) Prove by induction that $\left[\hat{x}^n, \hat{p}_x\right] = ni\hbar x^{n-1}$.

2. (a) For a Compton scattering, show that the change in wavelength of the scattered radiation is $\Delta\lambda = \frac{h}{m_0 c}[1-\cos\phi]$, where m_0 is the rest mass of electron, ϕ is the scattering angle of photon and h is Planck's constant.

(b) Why Compton effect cannot be observed with visible light?

(c) Both photoelectric and Compton effect are due to the action of photons on electrons, but the two effects are not same. Explain this.

(d) Light of wavelength 4500 Å ejects photoelectrons from a sodium surface (work function 2.3 eV). The stopping potential is experimentally found to be 0.46 volts. Calculate the value of Planck's constant. 6+2+2+2

3. (a) State and explain Heisenberg's uncertainty principle.

(b) Calculate the uncertainty in the momentum of a proton which is confined to a nucleus of radius equal to 10^{-13} cm. From this result, estimate the kinetic energy of the proton inside the nucleus and the strength of the nuclear interaction. What would be the kinetic energy for an electron if it had to be confined within a similar nucleus? What does this imply about the existence of electrons inside nuclei?

(c) An electron has a speed 500 m/s with an accuracy of 0.005%. Calculate the uncertainty in the position of the electron.

(d) A particle is described by the following wave function

$$\psi = 0 \text{ for } x < 0$$
$$= \sqrt{2}e^{-\frac{x}{L}} \text{ for } x \geq 0,$$

where L = 1nm. Calculate the probability of finding the particle in the region $x \geq$ 1nm.

2+(2+1+2+1)+2+2

4. A particle of mass m is in an infinite square well with boundaries at $x = 0$ and $x = a$. The wave function is given by

$$\psi(x) = \begin{cases} A\sin\frac{n\pi x}{a}, & \text{if } 0 \leq x \leq a \\ 0, & \text{otherwise, where } n = 1, 2, 3, \cdots \end{cases}$$

(a) Calculate the constant A.

(b) Calculate $\langle x \rangle, \langle x^2 \rangle, \langle p \rangle, \langle p^2 \rangle$.

(c) Calculate the uncertainties in position (Δx) and momentum (Δp).

(d) Check that the uncertainty principle is satisfied. 2+(2+2+2+2)+1+1

5. (a) Prove that if the linear operators $\hat{A}$ and $\hat{B}$ have a common complete set of eigenfunctions, then $\hat{A}$ and $\hat{B}$ commute.

(b) The parity operator $\hat{P}$ is defined as $\hat{P}\psi(x) = \psi(-x)$, where $\psi(x)$ is a wave function. Show that the eigenvalues of the parity operator are +1 and −1.

(c) If $H = \frac{p^2}{2m} + V(x)$, show that $\left[\hat{x},\left[\hat{x},\hat{H}\right]\right] = -\frac{\hbar^2}{m}$. 4+4+4

6. (a) Consider the operator $\hat{Q} = \frac{d^2}{d\varphi^2}$, where φ is the usual azimuthal angle in polar coordinates and the functions, f, are subject to the equation $f(\varphi) = f(\varphi + 2\pi)$.

(i) Prove that $\hat{Q}$ is Hermitian.

(ii) Find its eigenfunctions and eigenvalues.

(iii) Are the eigenvalues of $\hat{Q}$ degenerate?

(b) Suppose that $f(x)$ and $g(x)$ are two eigenfunctions of an operator $\hat{Q}$, with the same eigenvalue q. Show that any linear combination of $f(x)$ and $g(x)$ is itself an eigenfunction of $\hat{Q}$, with eigenvalue q. (3+4+2)+3

7. (a) An operator $\hat{A}$, representing observable A, has two (normalized) eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two (normalized) eigenstates φ_1 and φ_2, with eigenvalues b_1 and b_2. The eigenstates are related by $\psi_1 = \frac{3\varphi_1 + 4\varphi_2}{5}$, $\psi_2 = \frac{4\varphi_1 - 3\varphi_2}{5}$.

(i) Observable A is measured and the value a_1 is obtained. What is the state of the system (immediately) after this measurement?

(ii) If B is now measured, what are the possible results and what are their probabilities?

(iii) Right after the measurement of B, A is measured again. What is the probability of getting a_1?

(b) Show that the expectation value of an anti-Hermitian operator is either imaginary or zero.

(c) Show that the commutator of two Hermitian operators is anti-Hermitian. (1+3+2)+3+3

Please Turn Over

8. (a) Starting from the time-dependent Schrödinger equation deduce the equation of continuity for a particle moving in one dimension. What are stationary states? Why are they so called?

(b) Write down the time independent Schrödinger equation for a simple harmonic oscillator of mass m and frequency of oscillation ω_0. The eigenfunction of the Hamiltonian operator for the ground state is

$$\psi_0 = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{\alpha^2 x^2}{2}}, \text{ where } \alpha = \left(\frac{m\omega_0}{\hbar}\right)^{1/2}$$

(i) Find the energy eigenvalue corresponding to the ground state.

(ii) What is the most probable position of the particle in the ground state?

(4+2+1)+(1+3+1)

9. An electron with a kinetic energy of 10 eV is moving from left to right along the x-axis. The potential energy is $V = 0$ for $x < 0$ and $V = 20$ eV for $x > 0$.

(a) Write down the Schrödinger equation for $x < 0$ and $x > 0$.

(b) Show that the reflection coefficient is unity.

(c) Sketch the solutions in the two regions.

(d) What is the wavelength for $x < 0$?

(e) Is there any possibility of finding the electron in the region $x > 0$? How do you reconcile your answer with (b) above? Explain. 2+4+1+2+3