2025

STATISTICS — HONOURS

Paper : DSCC-2

(Descriptive Statistics - II and Probability - II)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Give a rough sketch of a scatter plot indicating a correlation coefficient near about – 0.5.

(b) For two uncorrelated variables x and y with equal variances, what is the correlation coefficient between $\left(\frac{x}{2}-y\right)$ and $\left(x-\frac{y}{2}\right)$?

(c) Suggest a set of few paired values of two related variables (x, y) for which their correlation coefficient is zero. Also state the relation between the two variables.

(d) Explain how the sign of the correlation coefficient is related to that of the two regression coefficients.

(e) Define a random variable.

(f) For what value of n can the following function be a probability mass function (p.m.f.)?

$$f(x)=\begin{cases}\frac{t}{55} & \text{if } x=t; t=1,2,...,n \\ 0 & \text{otherwise.}\end{cases}$$

(g) If the coefficient of variation of a symmetric binomial distribution is $\frac{1}{3}$, what would be the number of trials?

(h) If $F(x)$ is the cumulative distribution function (c.d.f.) of some random variable X, state whether the following are correct or not, with appropriate reasons thereof :

$$F(x) = -0.25, \quad F(x) = 0, \quad F(x) = 0.79, \quad F(x) = 1.05.$$

2. Answer ***any four*** questions : 5×4

(a) Give examples of :

(i) Two variables having no relationship at all.

(ii) For two related variables, one of them depends on the other but the opposite is not true.

(iii) Either of the two related variables can be treated as independent, the other dependent.

(b) The pressure (p) and volume (v) of a gas are known to be related by an equation of the form pv^{γ} = constant. Indicate how you would proceed to find the value of γ by fitting a straight line to set of observed values of p and v, taking p to be the independent variable.

(c) Suppose two judges rank n individuals based on their performance in some event, independently of one another. Derive an expression for the Spearman's rank correlation coefficient, when the rankings for one judge bears some ties of same length while those for the other judge do not bear a single tie.

(d) Define the cumulative distribution function of a random variable and state its important properties.

(e) Give an example of a discrete uniform distribution. Write down its probability mass function. Also find its cumulative distribution function.

(f) Derive the coefficient of variation of a Poisson distribution with parameter λ.

3. Answer ***any three*** questions :

(a) (i) What is *intra-class* correlation? How is it different from inter-class correlation?

(ii) Suppose a variable is measured for all the members of n families, the i-th family having t_i members, $i = 1, 2, ..., n$. Deduce the expression for the intra-class correlation coefficient r_I in this case. For $t_1 = t_2 = ... = t_n = t$, show that r_I lies between $-\frac{1}{t-1}$ and 1, discuss the cases $r_I = -\frac{1}{t-1}$ and $r_I = 0$. (2+2)+(5+4+2)

(b) Define Kendall's measure (τ) of association between two variables. Show that the numerical value of τ remains unchanged by monotonic transformation of variables. Show that τ is a special type of product moment correlation coefficient. Discuss on the cases $\tau = \pm 1$. Also establish that the numerical value of τ cannot exceed unity. 3+3+4+3+2

(c) (i) Define "odds ratio". Show that sample odds ratio does not change when both cell counts within any row (or column) are multiplied by a nonzero constant. Discuss the implication of the above result using a real-life example.

(ii) Describe one measure of association between two attributes. Discuss merits and demerits of this measure. (2+5+2)+(2+2+2)

(d) (i) Show that the cumulative distribution function $F(x)$ of a random variable X, with mean μ and variance $\sigma^2 (> 0)$, satisfies the inequalities :

$$F(x) \leq \frac{\sigma^2}{\sigma^2 + (x-\mu)^2} \quad \text{for } x \leq \mu$$

$$F(x) \geq \frac{(x-\mu)^2}{\sigma^2 + (x-\mu)^2} \quad \text{for } x \geq \mu.$$

(ii) Suppose X is a continuous random variable with distribution function $F(x)$ which satisfies $F(x) + F(-x) = 1$ for all x. If $E(X)$ exists, show that $E(X) = 0$. (5+5)+5

(e) (i) There are two coins with chance of Head 0.3 and 0.4, respectively. One of them is selected at random and tossed n times. Find the mean and variance of the number of Heads realized in this game.

(ii) Let X be a random variable with p.d.f.

$$f(x) = \begin{cases} ce^{-x} & x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the constant C. Find the distribution function of X, and hence find its mean.

(iii) Find the mean of a theoretical distribution having probability density function

$$f(x) = \frac{h}{\sqrt{\pi}} \exp[-h^2(x-a)^2], \text{ for } -\infty < x < \infty,$$

where $h > 0$. 6+(2+2+2)+3
