

2025

ECONOMICS — HONOURS

Paper : DSCC-12

(Econometrics - I)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group - A

1. Answer *any ten* questions :

2×10

(a) What is the most salient difference between economic model and econometric model?

(b) State whether the following statements are true or false :

(i) The conditional and unconditional means of a random variable are the same thing.

(ii) Even though the disturbance term in the CLRM is not normally distributed, the OLS estimates are still unbiased.

(iii) The t test requires that the sampling distribution of β_1 and β_2 follow normal distribution.

(iv) If there is no intercept term in the regression model, then the estimated error term will sum to zero,

(c) Consider the following regression results and fill in the blanks :

$$(\text{Consumption})_i = 24.4545 + 0.5091 (\text{Income})_i$$

$$SE = (6.4138) (?)$$

$$t = (?) (14.2605)$$

(d) The following ANOVA table is provided :

ANOVA Table

Source of variation	Sum of Squares	Degrees of Freedom
Regression	10,357	8
Residual	23,311	168

Determine the value of F -statistic.

(e) In a two variable PRF,

$$E(Y | X_i) = \beta_1 + \beta_2 X_i.$$

If the slope coefficient is zero, then what does that mean?

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- (f) In a simple linear regression model if all the observations of the independent variable are increased by a constant k , then how will the slope coefficient be affected?
- (g) Consider the following regression equation :

$$\hat{Y} = 3.6 + 0.75X$$

$$SE = (2.09) (0.256)$$

$$t = 2.930$$

$$n = 10$$

Calculate r .

- (h) Determine the type of data represented by the following variables :
- Population of 26 states of India in the year 2021.
 - Consumer Price Index in seven industrial countries for the period 1973-1997.
- (i) State the Gauss-Markov theorem.
- (j) What do you mean by forecast error?
- (k) What is meant by a first order autoregressive model?
- (l) In the simple linear regression model $y = \beta_0 + \beta_1 X + u$, suppose that $E(u) \neq 0$. Letting $\alpha_0 = E(u)$, show that the model can always be rewritten with the same slope, but a new intercept and error, where the new error has a zero expected value.
- (m) Distinguish between 'error' and 'residual' in the context of an econometric model.
- (n) What is the difference between ex-ante and ex-post forecast?
- (o) What is the difference between a deterministic relation and a stochastic relation?

Group - B

Answer *any five* questions.

2. The correlation between yield and rainfall is given by $r = 0.6$. However, temperature correlates both with yield ($r = 0.5$) and rainfall ($r = 0.8$). Find the correlation between yield and rainfall eliminating the effect of temperature. 5
3. (a) State one reason behind heteroskedasticity.
- (b) What are its consequences?
- (c) Mention a test for heteroskedasticity.
- (d) Which distribution does this test statistics follow? 1+2+1+1

4. Consider the following regression equation :

$$\hat{Y}_t = 29.5192 + 0.7136 X_t$$

$$SE = (4.1180) (0.0512)$$

$$r^2 = 0.9584, \text{ Durbin Watson statistics } (d) = 0.1229, n = 20.$$

(Given : Corresponding to 5% level of significance, $n = 20$ and number of regressors = 1, the critical d values are $d_L = 1.201$ and $d_U = 1.411$)

- (a) On the basis of the given information comment whether autocorrelation is present, and if so, what is the nature of autocorrelation?
(b) State two drawbacks of Durbin-Watson test. 3+2

5. Consider the following regression equation :

$$\text{income} = \beta_0 + \beta_1 \text{hw} + \beta_2 \text{ex},$$

where hw = number of hours worked; ex = years of experience.

If $\text{ex} = \alpha_0 + \alpha_1 \text{hw}$, then

- (a) What problem will you face in estimating the regression equation?
(b) State any one method that solves this problem. 3+2

6. (a) Consider the model $y = e^{\alpha + \beta x + u}$. Is it a linear regression model? Why or why not?

- (b) Consider the following estimated regression equation : $Y_i = \hat{\alpha} + 1.5X_i + e_i$, where the estimated standard error of the slope coefficient is 0.5. It has further been given that $r^2 = 0.5$, $\bar{x} = 10$, $\bar{y} = 25$ and $\sum y_i^2 = 6895$. Find the sample size (n), Total Sum of Squares (TSS) and estimated error variance. 2+3

7. (a) In a two-variable classical linear regression model, show that Total Sum of Squares (TSS) is equal to the sum of Explained Sum to Squares (ESS) and Residual Sum of Squares (RSS).

- (b) In the regression $Y_i = \beta_0 + \beta_1 X_i + u_i$ suppose we multiply each X -value by a constant 2. Will it change the residuals and the fitted values of Y ? What will happen if we add a constant 2 to each X -value? 2+3

8. Two variables Y and X are believed to be related by the following stochastic equation :

$$Y_i = \beta_0 + \beta_1 X_i + u_i,$$

where u_i is the usual random disturbance term with zero mean and constant variance σ^2 . You are given the following information : $n = 8$, $\sum X = 24$, $\sum X^2 = 75$, $\sum Y = 108$, $\sum Y^2 = 1620$ and $\sum XY = 343.5$.

- (i) Find the estimators of β_0 and β_1 ; (ii) Find the value of coefficient of determination; (iii) Test whether the slope coefficient is significantly different from zero at 1% level of significance (given $t_{0.005, 6} = 3.70$). 2+1+2

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9. Consider the regression model $Y_i = \beta X_i + u_i$ under the Gauss-Markov Assumptions. Derive the OLS estimator of β and show that it is unbiased. 3+2

Group - C

Answer *any three* questions.

10. (a) Define coefficient of determination (R^2). What role does it play in econometric analysis?
 (b) Consider the following estimated regression equation :

$$\log(G) = -10.477 - 0.387 \log(P_G) + 2.472 \log(Y)$$

$$SE = \quad (5.67) \quad (1.16) \quad (20.04)$$

$$\bar{R}^2 = 0.969; n = 14$$

where G = per capita consumption of gasoline; P_G = retail gasoline price; Y = per capita disposable income at constant prices.

- (i) What does the coefficient of Y stand for? Is it significant at 1% level?
 (ii) What is the price elasticity of demand for per capita gasoline consumption? Discuss the sign and magnitude of this coefficient.
 (iii) Determine the value of R^2 from the given value of \bar{R}^2 .

$$[\text{Given } t_{0.005, 11} = 3.106] \quad (1+2)+(2+3+2)$$

11. (a) Write a short note on importance of dummy variable in econometric analysis.
 (b) Consider the following estimated regression equation :

$$\hat{Y}_i = 22.008 - 0.002W - 2.76D_{2i} + 3.280D_{3i} + 0.415EPA$$

$$SE = \quad (0.001) \quad (0.708) \quad (1.413) \quad (0.097)$$

$$\bar{R}^2 = 0.82; n = 33$$

where Y = miles per gallon

W = weight of the car (pounds)

$D_{2i} = 1$ for automatic transmission

= 0 otherwise

$D_{3i} = 1$ for diesel

= 0 otherwise

EPA = mileage rating by EPA (Environmental Protection Agency)

- (i) Interpret the regression coefficient of D_2 in the above model. Is it significant at 1% level?
 (ii) Interpret the regression coefficient of D_3 in the above model. Is it significant at 1% level?

$$[\text{Given } t_{0.005, 28} = 2.763] \quad 5+(2\frac{1}{2}+2\frac{1}{2})$$

12. (a) State the assumptions of classical linear regression model.
 (b) Two variables Y and X are believed to be related by the following stochastic equation :

$$Y = \alpha + \beta X + U,$$

where U is the usual random disturbance term with zero mean and constant variance (σ^2).

To check this relationship one researcher takes a sample of size 8 and estimates β with OLS. Another researcher takes another sample of size 8 and estimates β with OLS. The data they used and the results they obtained are as follows :

Researcher I	
Y	X
4.0	3
4.5	3
4.5	3
3.5	3
4.5	4
4.5	4
5.5	4
5.0	4

$$\hat{Y} = 1.857 + 0.75X$$

$$SE = (1.20) (0.339)$$

$$r^2 = 0.45; \hat{\sigma} = 0.48$$

Researcher II	
Y	X
2.0	1
2.5	1
2.5	1
1.5	1
11.5	10
10.5	10
10.5	10
11.5	10

$$\hat{Y} = 1.5 + 0.97X$$

$$SE = (0.27) (0.038)$$

$$r^2 = 0.99; \hat{\sigma} = 0.48$$

- (i) Can you explain why the SE of β for the first researcher is larger than the SE of $\hat{\beta}$ of the second researcher?
 (ii) Which estimated model according to you is better and why? 5+(3+2)
13. (a) Explain mathematically and intuitively what would happen if you tried to fit a regression equation when all the values of the explanatory variables in the sample are the same.

- (b) Consider the following regression result : $\hat{Y}_i = 2.20 + 0.104X_{1i} + 3.48X_{2i}$.

$$p\text{-value } 0.000 \quad 0.001 \quad 0.044$$

$$ESS = 112.5, \quad RSS = 19.5, \quad n = 10$$

- (i) Which of the estimated slope coefficients are statistically significant at 1% level of significance?
 (ii) Calculate the value of R^2 and adjusted- R^2 .
 (iii) Calculate the F -statistic and interpret it. [Given $F_{0.01, 2, 7} = 9.55$; $F_{0.01, 7, 2} = 99.36$]. 5+5
14. (a) A regression of average daily earnings (E) measured in rupees on age (A) measured in years using a random sample of workers yields

$$\hat{E} = 696.7 + 9.6 A; \quad R^2 = 0.023, \quad RSS = 1542.2$$

- (i) What are the units of RSS and R^2 ?
 (ii) What is the predicted earning of a 30 years old worker?
 (iii) What is the average increase in earning for one year increase in age?

Please Turn Over

(b) Consider the following model :

$$\hat{Y}_i = -0.2610 - 2.3606 D_i + 0.8028 X_i \quad (D_i = 1 \text{ if female, } 0 \text{ otherwise})$$

$$SE = (1.1073) (0.4302) (0.08101) \quad R^2 = 0.2032; n = 528$$

Here Y = hourly wage and X = year of schooling.

- (i) Interpret the meaning of the coefficients in the above model.
 - (ii) Test in the context of the above model whether there is any difference in hourly wage between males and females.
 - (iii) Test in the context of the above model whether hourly wage is significantly related to years of schooling. [Given : $t_{0.01, 525} = 2.576$]. (1+1+1)+(3+2+2)
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