2025

MATHEMATICS — HONOURS

Paper : DSCC-12

(Mechanics - II)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Statics - II]

(Marks : 28)

1. Answer ***any four*** questions : 7×4

(a) A rough wire which has shape of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is placed with its x-axis vertical and y-axis horizontal. If μ be the coefficient of friction, find the depth below the highest point of the position of limiting equilibrium of a bead which rest on the wire.

(b) If a system of coplanar forces acting on a rigid body be in equilibrium and the body undergo a slight displacement consistent with the geometrical conditions of the system, then prove that the algebraic sum of the virtual works is zero.

(c) A rod AB is movable about a joint at A, and to B is attached a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. Prove by the principle of virtual work that the horizontal force necessary to keep the ring at rest is

$$\frac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)},$$

where W is the weight of the rod and α, β are respectively the inclinations of the rod and the string to the horizontal.

(d) A lamina in the form of a cycloid, whose generating circle is of radius a, rests on the top of another cycloid whose generating circle is of radius b, their vertices being in contact and their axes vertical. If h be the height of the centre of gravity of the upper cycloid above its vertex, show that the equilibrium is stable, only if $h<\frac{4ab}{(a+b)}$.

(e) A solid sphere rests inside a fixed rough hemispherical bowl of twice its radius. Show that, however large a weight is attached to the highest point of the sphere, the equilibrium is stable.

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(f) Let X, Y, Z and L, M, N denote respectively the algebraic sum of the components of a system of forces and their moments w.r.t. Cartesian axes Ox, Oy, Oz passing through any base point O. Show that $X^2 + Y^2 + Z^2$ and $LX + MY + NZ$ are invariant whatever be the base point of the axes.

(g) Two forces act, one along the line $y = 0$, $z = 0$ and the other along the line $x = 0$, $z = c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2 + y^2)z = cy^2$.

Group - B

[Dynamics of a Particle - II]

(Marks : 15)

2. Answer ***any one*** question :

(a) A particle describes a path which is nearly a circle about a centre of force $\varphi(u)$ per unit mass, u being the reciprocal of the distance from the centre of force. Find the condition that this may be a stable motion. Hence find the apsidal angle. 6+2

(b) Deduce the velocity and acceleration components of a particle referred to a set of rectangular axes, rotating in their plane about a line perpendicular to the plane and passing through their meeting point. 8

(c) If a rocket, originally of mass M, throws off at every unit of time a mass eM with relative velocity V and if M' be the mass of the case etc., then show that it cannot rise at once unless $eV > g$, nor at all unless $\frac{eMV}{M'} > g$.

If it just rises vertically at once, then show that its greatest velocity is $V \log \frac{M}{M'} - \frac{g}{e}\left(1 - \frac{M'}{M}\right)$. 2+2+4

3. Answer ***any one*** question :

(a) Define degrees of freedom and configuration space of a mechanical system. Deduce the equation of motion of the centre of mass of a system of particles. 2+5

(b) State the conservation theorem for total angular momentum of a system of particles. Prove that total angular momentum of a system of particles with respect to the centre of mass is conserved. 2+5

(c) State and explain the law of conservation of energy for a system of particles. Find the total energy of a system of particles under conservative forces. 2+5

Group - C

[Dynamics of Rigid Body]

(Marks : 32)

4. Answer ***any four*** questions :

(a) Show that the equation of the momental ellipsoid at a point on the rim of a hemisphere is $8x^2 + 28(y^2 + z^2) - 15xz = \text{constant}$. 8

(b) State D'Alembert's principle. A plank of mass M and length a, is initially at rest along a line of greatest slope, of a smooth plane inclined at an angle α to the horizon and a man of mass M_1, starting from the upper end walks down the plank so that it does not move. Show that he will reach the other end in time $\sqrt{\dfrac{2M_1 a}{(M+M_1)g\sin\alpha}}$. 2+6

(c) Using D'Alembert's principle deduce the equations of motion of the centre of inertia of a rigid body and the equation of motion relative to the centre of inertia. Show that the above two motions are not dependent. 5+3

(d) A uniform rod of mass m and length $2a$, can turn freely about a fixed axis; show that the least angular velocity with which it must be started from the lowest position so that it may just make complete revolution is $\sqrt{\dfrac{3g}{a}}$. Also show that with this angular velocity, time to describe an angle θ is $\sqrt{\dfrac{4a}{3g}}\log\tan\left(\dfrac{\pi}{4}+\dfrac{\theta}{4}\right)$. 4+4

(e) In two-dimensional motion of a rigid body, prove with usual notations that its kinetic energy is

$$\frac{1}{2}MV^2 + \frac{1}{2}MK^2\dot{\theta}^2 .$$ 8

(f) Define 'Impulse of a force'. Obtain the equation of motion in two dimensions of a rigid body moving under impulsive forces. 1+7

(g) AB, BC are two equal similar rods freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB, show that the resulting velocity of A is $3\frac{1}{2}$ times that of B. 8