2025

MATHEMATICS — HONOURS

Paper : DSCC-9

(Probability and Statistics)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Probability]

(Marks : 45)

1. Answer ***any five*** questions : 3×5

(a) Consider events A and B such that

$$P(A)=\frac{1}{4},\ P(B\mid A)=\frac{1}{2},\ P(A\mid B)=\frac{1}{4}.$$

Find $P(\bar{A}\mid\bar{B})$.

(b) Let $A=[a_{ij}]$ be a square matrix of order 2 with entries either 0 or 1. Let X be the event that A is an invertible matrix, then find the probability $P(X)$.

(c) The probability of hitting a target is 0.0005. How many times one has to fire so that the probability of hitting a target at least once is more than 80%?

(d) Calculate the variance of normal distribution.

(e) Consider the distribution function of X given by

$$F(X)=\begin{cases}0 & \text{for } x<0\\ 1-\frac{1}{4}e^{-x} & \text{for } x\geq 0\end{cases}$$

Determine $P(X>0)$.

(f) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant K $(0<K<1)$ is $K(1-\log_e K)$.

(g) Find the characteristic function of a Poisson μ-variate.

(h) Show that X and $Y-\rho\frac{\sigma_y}{\sigma_x}X$ are uncorrelated random variables, where the symbols have their usual meaning.

Please Turn Over

2. Answer ***any five*** questions :

(a) From an urn containing 3 white and 5 black balls, 4 balls are transferred to an empty urn. From the second urn 2 balls are drawn and they happen to be white. What is the probability that the third ball drawn from the same urn will be white? 6

(b) Find the probability of obtaining a sum of 14, with results of throwing 3 dice. 6

(c) Define distribution function of a random variable and show that it is a monotonic increasing function. If X is a random variable and a is a real number such that $P(X = a) > 0$, show that the distribution function of X is discontinuous at the point a. 2+2+2

(d) If X is a $N(0, 1)$ variate, then find the distribution of

(i) $Y = \frac{1}{2}X^2$

(ii) $Y = e^X$. 4+2

(e) In the equation $x^2 + 2x - q = 0$, q is a random variable uniformly distributed over the interval $(0, 2)$. Find the distribution function of the larger root. 6

(f) The density function of a two-dimensional random variable (X, Y) is given by

$$f(x,y) = \begin{cases} Kxy(x+y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

(i) the value of K.

(ii) $P\left(\frac{1}{3} < Y \le \frac{2}{3} \middle| \frac{1}{2} < X \le \frac{3}{4}\right)$. 2+4

(g) How many independent Bernoulli trials are required to ensure with a probability not less than 0.975, the validity of the inequality $\left|\frac{1}{n}X_n - p\right| < 0.1$, where X_n is the number of successes in n trials, where $p = \frac{1}{2}$ is the probability of a success in one trial? 6

(h) A continuous random variable X is distributed over the interval $(0, 1)$ with *pdf* $ax^2 + bx$, where a and b are two constants. If $E(X) = 0.5$, then find the values of a and b. Hence find the third central moment of X. 3+3

Group - B

[Statistics]

(Marks : 30)

3. Answer ***any three*** questions : 3×3

(a) Define an estimator. When is an estimator called unbiased and consistent?

(b) Find a 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample (2.3, −0.2, −0.4, −0.9).

$$\left[\text{Given}: \frac{1}{\sqrt{2\pi}} \int_{1.960}^{\infty} e^{-t^2/2}\, dt = 0.025\right].$$

(c) Given the probability density function

$$f(x,\theta) = \frac{1}{\theta} \quad 0 \le x \le \theta.$$

Find the probability of Type-II error to test the hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ in the critical region $1 \le x \le 1.5$. Also find the power of the test.

(d) A company sells ice cream as packs of 500 gm. A sample of 16 packs is taken for inspection and sample mean is found to be 480 gm with a standard deviation of 30 gm. Does the sample differ significantly from the intended weight of 500 gm?

[Given $t_{0.025,\,15} = 2.13$].

(e) If the density function of a population is given by $\frac{x^{l-1} e^{-\frac{x}{\alpha}}}{\alpha^l \Gamma(l)}$, $0 < x < \infty$, l being a known constant; estimate the parameter α by the method of maximum likelihood.

4. Answer ***any three*** questions :

(a) What is meant by the confidence interval for a parameter of a distribution? Find the confidence interval for the variance of a normal (m, σ) population on the basis of a random sample of size n and a confidence coefficient $1 - \alpha$. 2+5

(b) Nine patients to whom a certain drug is administered, registered the following rise in blood pressure in mm of Hg : 3, 7, 4, −1, −3, 6, −4, 1, 5. Test the hypothesis that the drug did not raise blood pressure at 10% significance level, assuming that the sample is chosen from a normal population. Given $P(t > 1.86) = 0.05$ for eight degrees of freedom. 7

(c) A die is thrown 150 times with the following data :

No. turned up	1	2	3	4	5	6
Frequency	19	23	28	17	32	31

Are the data consistent with the hypothesis that the die is unbiased? [Given $\chi^2_{0.05,5} = 11.07$] 7

Please Turn Over

(d) (i) A bivariate sample of size 11 gave the result $\bar{x} = 7, S_x = 2, \bar{y} = 9, S_y = 4$ and $r = 0.5$. It was found that one of the sample value ($x = 7$, $y = 9$) was inaccurate and rejected. How that value of r will be affected by the rejection?

(ii) Show that correlation coefficient is independent of change of origin. 5+2

(e) Fit a second degree parabola $y = a + bx^2$, to the following data by the method of least square.

x	0	1	2	3	4
y	1	5	10	22	38

Estimate y when $x = 2.5$. 6+1