

2025

## MATHEMATICS — MINOR

Paper : MN-3

(Ordinary Differential Equations and Group Theory)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**All the notations and symbols have their usual meanings.*

## Group - A

(Ordinary Differential Equations)

Answer *any nine* questions.

1. (a) Solve :  $\sqrt{1-x^2} dy + \sqrt{1-y^2} dx = 0$ .

(b) Show that the general solution of the equation  $\frac{dy}{dx} + Py = Q$  can be written as  $y = K(u - v) + v$ , where  $K$  is a constant and  $u, v$  are its two particular solutions. 2+3

2. Define Integrating factor of a differential equation and solve :  $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ . 1+4

3. Solve :  $x \frac{dy}{dx} + y = y^2 \log x$ . 5

4. Reduce the differential equation  $x^2 p^2 + py(2x + y) + y^2 = 0$  to Clairaut's form by the substitution  $y = u$ ,  $xy = v$  and solve it to find general solution and singular solution. 1+2+2

5. Solve  $(D^2 + 1)y = 3\cos^2 x$ , where  $D \equiv \frac{d}{dx}$  by the method of  $D$ -operator. 5

6. Solve by the method of variation of parameters :  $\frac{d^2 y}{dx^2} + 9y = \sec 3x$ . 5

7. Solve by the method of undetermined coefficients  $\frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 6y = (x - 2)e^x$ . 5

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8. Solve and find the singular solution of  $xp^2 - 2py + 4x = 0$   $\left( p = \frac{dy}{dx} \right)$ . 5

9. Solve  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = 0$ , given that  $\left( x + \frac{1}{x} \right)$  is one integral (or solution). 5

10. Solve the following Simultaneous Linear differential equations : 5

$$(D - 1)x + Dy = 2t + 1$$

$$(2D + 1)x + 2Dy = t, \text{ where } D \equiv \frac{d}{dt}.$$

11. Show that the family of confocal conics  $\frac{x^2}{(a^2 + \lambda)} + \frac{y^2}{(b^2 + \lambda)} = 1$  is self-orthogonal,  $\lambda$  is a parameter. 5

12. Solve :  $\frac{d^2y}{dx^2} + 2 \sin x \frac{dy}{dx} + 2y \cos x = 0$ . 5

13. Solve the equation :  $(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$ . 5

14. Solve the simultaneous equation :  $\frac{x dx}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}$ . 5

15. Solve the differential equation :  $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$   $\left[ D \equiv \frac{d}{dx} \right]$ . 5

16. Find an integrating factor of the differential equation  $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$  and hence solve it. 2+3

### Group - B

#### (Group Theory)

Answer *any six* questions.

17. Prove that the set  $Q$  of all rational numbers other than 1 under the binary operation '\*' defined by  $a * b = a + b - ab$  forms an Abelian group for all  $a, b \in Q - \{1\}$ . 5

18. Define order of a group. Also, explain the order of an element in a group and hence prove that each element of a finite group is of finite order. 1+1+3

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19. Let  $G$  be a group and  $H, K$  are two subgroups of  $G$ . Prove that  $H \cap K$  is also a subgroup of  $G$ . Is  $H \cup K$  also a subgroup? Justify your answer. 3+1+1
20. Let  $G = \left\{ \begin{bmatrix} a & 0 \\ b & 1 \end{bmatrix} : a, b \in \mathbb{R}, a \neq 0 \right\}$ . Show that  $G$  becomes a group under usual matrix multiplication. 5
21. If  $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$  be an element of  $S_7$ , then find  $\alpha^{-1}$ . Is it an even permutation? 3+2
22. Define subgroup of a group  $G$ . Let  $G$  be a multiplicative group and  $H$  be its subset defined by :  
 $H = \{x : x \in G \text{ \& } xa = ax, \forall a \in G\}$ .  
Prove that  $H$  is a subgroup of  $G$ . 1+4
23. Prove that every group of prime order is cyclic. 5
24. Show that the 8th roots of unity forms a cyclic group. 5
25. Prove that every cyclic group is a commutative group. Is the converse true? Justify your answer. 3+1+1
26. Prove that for a cyclic group  $G$ , there exists an element  $a$  in  $G$  such that  $o(a) = o(G)$ . 5