2025

PHYSICS — HONOURS

Paper : DSCC-11

(Quantum Mechanics)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions from the rest, taking at least ***two*** from **Group – A** and at least ***one*** from **Group – B**.

1. Answer ***any five*** questions : 3×5

(a) Show that if two operators commute they have common eigenstates.

(b) Show that wave function of a particle under central potential in spherical polar coordinates has definite parity.

(c) In one-dimensional harmonic oscillator the states are defined by $|1\rangle = a^{\dagger}|0\rangle$ (notations are usual).

Find $a|1\rangle$ and $a^{\dagger}|1\rangle$.

(d) A 3D anisotropic harmonic oscillator has frequencies $\omega_x = \omega$, $\omega_y = 2\omega$ and $\omega_z = 3\omega$. Determine the energy (in units of $\hbar\omega$) and the degeneracy (g) of the ground state.

(e) Prove that the Pauli matrices satisfy the cyclic commutation relation : $[\sigma_x, \sigma_y] = 2i\sigma_z$.

(f) If the number density of a free electron gas in 3D is increased eight times, show how its Fermi temperature changes.

(g) Plot the Fermi-Dirac distribution functions for $T = 0$ and $T \neq 0$. Show the position of Fermi energy in the plot.

(h) A system of massless bosons is in equilibrium at a temperature T. Show that the density of particles is proportional to T^3.

Group – A

2. (a) For a 1D harmonic oscillator, define $a = \sqrt{\frac{m\omega}{2\hbar}}\,x + i\sqrt{\frac{1}{2m\hbar\omega}}\,p$. What is $a^{\dagger}$? Show that $[a, a^{\dagger}] = 1$.

(b) Show that the Hamiltonian $H = \left(a^{\dagger}a + \frac{1}{2}\right)\hbar\omega$.

(c) If ground state is given by $|0\rangle$, find the corresponding ground state wave function $\psi_0(x)$.

(1+4)+3+4

3. (a) Define $L_{\pm} = L_x \pm iL_y$. Show that

$$[L_z, L_+] = \hbar L_+$$
$$[L_+, L_-] = 2\hbar L_z.$$

(b) Show that $L_+L_- = L^2 - L_z^2 + i\hbar L_z$.

(c) Construct the matrix representation of L_x and L_y for the state $l = 1$. You may use

$L_+|lm\rangle = \sqrt{l(l+1) - m(m+1)}\,\hbar|l, m+1\rangle$. 4+4+4

4. Consider a particle of mass m, confined in an infinite rectangular box of Length L and $2L$ $(0 < x < L,\ 0 < y < 2L)$.

(a) Write the two-dimensional Schrödinger equation in Cartesian Coordinates (solution not required).

(b) Write the expression for the energy eigenvalues in terms of the relevant quantum numbers.

(c) Find the ground state energy.

(d) Write the ground state wave function (determination of normalization constant not required).

(e) Find the energy for the 1st excited state and check whether it is degenerate. 2+2+2+2+(2+2)

5. (a) Consider a two-dimensional space spanned by the orthonormal basis vectors $|1\rangle \equiv \begin{pmatrix}1\\0\end{pmatrix}, |2\rangle \equiv \begin{pmatrix}0\\1\end{pmatrix}$.

An operator $\hat{A}$ is given by $\hat{A} = 2|1\rangle\langle 1| + i|1\rangle\langle 2| - i|2\rangle\langle 1| + 2|2\rangle\langle 2|$.

Find the matrix representation for $\hat{A}$.

(b) Verify that $\hat{A}$ is hermitian.

(c) Find the eigenvalues and normalised eigenvectors.

(d) Consider a state $|n\rangle = \frac{1}{\sqrt{2}}[|1\rangle - |2\rangle]$

Calculate $\langle n|\hat{A}|n\rangle$. 2+2+5+3

6. (a) Consider the ground state wave function of the hydrogen atom $\psi_{100}(r, \theta, \phi) = Ae^{-r/a_0}$, where a_0 is a constant. Calculate where the chance of finding the electron is maximum. Also calculate $\langle r\rangle$ for this state.

(b) Calculate the maximum possible value (in units of $\hbar$) that can be measured for the z-component of the angular momentum $\left(\hat{L}_z\right)$ when the hydrogen atom is in the $n = 4$, $l = 3$ state. If the atom is in the 4f subshell, what is the maximum value for the total angular momentum squared, $\left\langle \hat{L}^2 \right\rangle$?

(c) The radial probability density $P(r)$ is given by $P(r) = r^2 \left|R_{n,l}(r)\right|^2$. Sketch the radial probability density $P(r)$ for the $n = 2$, $l = 0$ state (2s). Mention the number of radial nodes ($r \neq 0$) in this sketch. (3+2)+(2+2)+(2+1)

Group – B

7. (a) Write the principle of symmetry postulate for a system of identical particles. Explain the classification with examples.

(b) Write down the BE distribution formula, explaining all the symbols.

(c) Calculate the total energy of a photon gas in equilibrium using BE distribution. Calculate the pressure of such a gas. (2+2)+2+(3+3)

8. (a) Consider a system of non-interacting spin $\frac{1}{2}$ fermions at absolute zero temperature. Calculate Fermi energy, total energy and pressure of the gas.

(b) Calculate the no. of ways in which 3 identical spin $\frac{1}{2}$ fermions can be distributed between two non-degenerate energy levels. Repeat the calculation for 3 identical spin-0 bosons. 6+3+3

9. (a) Derive the Fermi-Dirac distribution function starting from the grand-canonical ensemble.

(b) Why is the chemical potential μ for photon gas zero?

(c) Describe the variation of the electronic specific heat of a metal with temp. (T) at very low-temperatures according to the F-D statistics. Compare your results with classical prediction for the specific heat capacity $\left(C_v = \frac{3}{2}R\right)$. 6+2+4

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