2025

MATHEMATICS — HONOURS

Paper : DSCC-1

(Calculus, Geometry and Vector Analysis)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Calculus]

(Marks : 20)

1. Answer ***any two*** questions : 2×2

(a) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$, show that $I_n + I_{n-2} = \frac{1}{n-1}$.

(b) Find the length of the curve $y = \log\sec x$ between $x = 0$ and $x = \frac{\pi}{3}$.

(c) Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$.

2. Answer ***any four*** questions :

(a) Determine the value of p and q for which $\lim_{x\to 0}\frac{x(1+p\cos x)-q\sin x}{x^3}$ exists and equal to 1. 2+2

(b) If $y = \cosh(\sin^{-1}x)$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+1)y_n = 0$. Hence find the value of y_n at $x = 0$. 3+1

(c) If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x\,dx$ and $n > 1$, prove that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find $\int_0^{\frac{\pi}{2}} x^4 \sin x\,dx$. 3+1

(d) Determine the length of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, measured from the origin to any point $\theta = \theta_1$. 4

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(e) Find the area in the first quadrant included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$. 4

(f) Find the area of the surface generated by revolving the parabola $y^2 = 4ax$ about x-axis bounded by the section $x = a$. 4

Group - B

[Geometry]

(Marks : 35)

3. Answer ***any two*** questions : 2½×2

(a) Find the point on the conic $r = \dfrac{21}{5 - 2\cos\theta}$, which has the smallest radius vector.

(b) Find the condition for which the straight line $lx + my + n = 0$ is a normal to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

(c) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z + 3 = 0$ as a great circle.

4. Answer ***any five*** questions :

(a) Reduce the equation $x^2 - 4xy + 4y^2 + 2x - 4y + c = 0$ to its canonical form and determine the type of the conic represented by it for different values of c. 6

(b) Prove that the locus of the point of intersection of two perpendicular tangents to the conic $\dfrac{l}{r} = 1 + e\cos\theta$, is $(e^2 - 1)r^2 - 2ler\cos\theta + 2l^2 = 0$. 6

(c) Show that the locus of the middle points of the normal chords of the parabola $y^2 = 4ax$ is $\dfrac{y^2}{2a} + \dfrac{4a^3}{y^2} = x - 2a$. 6

(d) Prove that the centres of the spheres which touch the lines $y = mx$, $z = c$ and $y = -mx$, $z = -c$ lie upon the conicoid $mxy + c(1 + m^2)z = 0$. 6

(e) Show that the plane $ax + by + cz = 0$ may cut the cone $xy + yz + zx = 0$ in perpendicular lines if $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 0$. 6

(f) Find the locus of the luminous point if the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ cuts circular shadow on the plane $z = 0$. 6

(g) Find the equations of the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point (1, 1, 1). Hence find the angle between these generators. 5+1

(h) Show that the normals to the ellipsoid $x^2 + 2y^2 + 3z^2 = 50$ at the points (6, 1, 2) and (6, –1, 2) lie in the plane $x - z = 4$. 6

Group - C

[Vector Analysis]

(Marks : 20)

5. Answer ***any two*** questions : 2×2

(a) Prove the identity $\left[\vec{a}\times\vec{b}\;\;\vec{b}\times\vec{c}\;\;\vec{c}\times\vec{a}\right]=\left[\vec{a}\;\vec{b}\;\vec{c}\right]^2$.

(b) If $\vec{\alpha}=t^2\hat{i}-t\hat{j}+(2t+1)\hat{k}$ and $\vec{\beta}=(2t-3)\hat{i}+\hat{j}-t\hat{k}$, where $\hat{i},\hat{j},\hat{k}$ have their usual meanings, then find $\left\{\vec{\alpha}\times\frac{d\vec{\beta}}{dt}\right\}$ at $t = 2$.

(c) If $\vec{r}=t^2\hat{i}-e^{\sin\left(t\frac{\pi}{2}\right)}$ for $t\in [-1, 1]$, then find $\int_{-1}^{1}\left(\vec{r}.\frac{d\vec{r}}{dt}\right)dt$.

6. Answer ***any four*** questions :

(a) If $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\frac{\sqrt{3}}{2}\vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$, are non-parallel. 4

(b) Find the vector equation of the plane passing through the points (1, 1, 1) and (2, –1, 3) and parallel to the vector $3\hat{i}-4\hat{j}+4\hat{k}$. 4

(c) Show that the perpendicular distance of any point $\vec{a}$ from the line $\vec{r}=\vec{b}+\lambda\vec{c}$ is $\left|\frac{\left(\vec{b}-\vec{a}\right)\times\vec{c}}{|\vec{c}|}\right|$. 4

(d) If $\vec{r}(t)=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$, then evaluate

(i) $\int_{1}^{2}\left(\vec{r}\times\frac{d^2\vec{r}}{dt^2}\right)dt$

(ii) $\int_{1}^{2}\left(\frac{1}{r}\frac{d\vec{r}}{dt}-\frac{dr}{dt}\frac{\vec{r}}{r^2}\right)dt$, where $r=|\vec{r}|$. 2+2

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(e) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\,\hat{k}$, where $\hat{i}, \hat{j}, \hat{k}$ have their usual meanings, find the value of $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$. 4

(f) For any two vectors $\vec{u}$ and $\vec{v}$, prove that $\left(1+|\vec{u}|^2\right)\left(1+|\vec{v}|^2\right) = (1-\vec{u}.\vec{v})^2 + \left|\vec{u}+\vec{v}+(\vec{u}\times\vec{v})\right|^2$. 4

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