2025

MATHEMATICS — HONOURS

Paper : DSCC-4

(Ordinary Differential Equations - I and Group Theory - I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

(Ordinary Differential Equations - I)

Answer ***any nine*** questions.

1. Find the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$. 5

2. Show that $\frac{1}{3x^3y^3}$ is an integrating factor of $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ and hence solve it. 2+3

3. Find the value of constant λ for which the differential equation $(2xe^y + 3y^2)dy + (3x^2 + \lambda e^y)dx = 0$ is exact. Hence solve the equation. 2+3

4. Solve : $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$. 5

5. Reduce the differential equation $y = 2px - p^2y$ to Clairaut's form by the substitutions $y^2 = Y$, $x = X$ and then obtain the complete primitive and singular solution, if any. 5

6. Solve $(D^3 + 3D^2 - 4)y = xe^{-2x}$ $\left(D \equiv \frac{d}{dx}\right)$. 5

7. Solve the following differential equation by the method of variation of parameters : 5

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

8. Solve the differential equation $(D^2 - 2D + 5)y = e^x\sin 2x$ by the method of undetermined coefficients. 5

9. Solve : $\left(x^2D^2 - xD + 4\right)y = \cos(\log x) + x\sin(\log x)$. 5

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10. Solve the following by changing the independent variables : 5

$$\left(1+x^2\right)^2 \frac{d^2y}{dx^2} + 2x\left(1+x^2\right)\frac{dy}{dx} + 4y = 0.$$

11. Solve the following differential equation by factorisation of operators : 5

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (1+x)e^x.$$

12. Solve the differential equation $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = e^x$, by showing that it is exact. 5

13. Solve : $y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = y^2 \log y$. 5

14. Solve for x and y :

$$\frac{dx}{dt} - 3x + 4y = e^{-2t}$$
$$\frac{dy}{dt} - x + 2y = 3e^{-2t}$$ 5

Group - B

(Group Theory - I)

Answer ***any six*** questions.

15. (a) In S_8, Let $\alpha = (1 \ 2 \ 3 \ 4 \ 5)$ and $\beta = (5 \ 6 \ 7)$. Find the order of the permutation $\alpha\beta^{-1}$.

(b) Find the image of the elements 3 and 4 if $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & & & 4 \end{pmatrix}$ is an odd permutation. 3+2

16. Let (G, o) be a semi-group and for $a, b \in G$, the equations $a \text{ o } x = b$ and $y \text{ o } a = b$ has a solution in G. Then prove that (G, o) is a group. 5

17. Prove that the set of matrices M given by, $M = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix}; x \in \mathbb{R}, x \neq 0 \right\}$ forms an abelian group under matrix multiplication. 5

18. (a) Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$, for all $a, b \in G$.

(b) State whether the set $G = \{a^m b^n : m, n \in \mathbb{Z}\}$ is a group under multiplication, where $a, b > 0$ are fixed real numbers. Justify your answer. 3+2

19. In a group (G, o), the elements a and b commute and $o(a)$ and $o(b)$ are prime to each other. Show that $o(a \, o \, b) = o(a) \, . \, o(b)$. 5

20. Prove that every cyclic group is an abelian group. Is the converse true? Justify your answer. 3+2

21. (a) Prove that centre of a group is a subgroup of the group.

(b) Find all elements of order 5 in the group $(\mathbb{Z}_{20}, +)$. 2+3

22. State and prove Lagrange's theorem. 1+4

23. Prove that every group of order less than 6 is commutative. 5