2025

MATHEMATICS — HONOURS

Paper : SEC-3

(Linear Programming and Rectangular Games)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 3×5

(a) Solve the following LPP by graphical method :

Minimize $Z = 4x_1 + x_2$

subject to $x_1 + 2x_2 \leq 3$

$4x_1 + 3x_2 = 6$

$3x_1 + x_2 \geq 3,\ x_1, x_2 \geq 0$

(b) Find a basic solution of

$$x_1 + 4x_2 - x_3 = 3$$
$$5x_1 + 2x_2 + 3x_3 = 4$$

with x_3 as non-basic variable.

(c) Prove that a hyperplane is a convex set.

(d) Prove that the set $X = \{(x, y) \in E^2 : y^2 \leq x\}$ is a convex set.

(e) Using simplex method, solve the following LPP :

Maximize $Z = 3x_1 + 9x_2$

subject to $2x_1 + 3x_2 \leq 6$

$2x_1 + 6x_2 \leq 6$

$x_1, x_2 \geq 0.$

Please Turn Over

(f) Find the dual of the following LPP :

Maximize $Z = x_1 - x_2 + 3x_3 + 2x_4$

subject to
$$x_1 + x_2 \geq -1$$
$$x_1 - 3x_2 - x_3 \leq 7$$
$$x_1 + x_3 - 3x_4 = -2$$

$x_1, x_2, x_3, x_4 \geq 0.$

(g) Obtain an intial basic feasible solution to the following Transportation problem using the matrix minima method :

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	

(h) Apply maximin and minimax principle to solve the game whose pay-off matrix is given below :

Player B

Player A	B_1	B_2	B_3
A_1	–1	–2	8
A_2	7	5	–1
A_3	6	0	12

Group - B

2. Answer ***any five*** questions :

(a) A furniture manufacturer plans to make only two products, chairs and tables, from his available resources. He has 400 board feet of wood and 450 man-hours. A chair requires 5 board feet of wood and 10 man-hours and yields a profit of ₹ 140.00, while one table requires 20 board feet of wood and 15 man-hours and yields a profit of ₹ 300. Pose a linear programming problem so that he can maximize his profit. 6

(b) (i) Is (2, 0, 0, 1, 0) a basic feasible solution of the system of equations
$$2x_1 + 2x_2 - x_3 + x_4 + 5x_5 = 5$$
$$4x_1 + 3x_2 - 3x_3 + 2x_4 - 3x_5 = 10?$$
Justify your answer.

(ii) (2, 4, 5) is a feasible solution of the set of equations
$$2x_1 - x_2 + 2x_3 = 10$$
$$x_1 + 4x_2 = 18$$
Reduce it to a basic feasible solution of the set. 2+4

(c) Prove that the objective function of a linear programming problem attains its optimal value (if it exists) at an extreme point of the convex set of feasible solutions. 6

(d) What do you mean by degenerate basic feasible solution?

Solve graphically to show that the LPP

Maximize $Z = 3x_1 + 9x_2$

subject to $x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4,\ x_1,\ x_2 \geq 0$

admits of a degenerate optimal basic feasible solution. 2+4

(e) Solve the following linear programming problem by simplex method : 6

Maximize $Z = 3x_1 + 5x_2 + 2x_3$

subject to $x_1 + x_2 + 2x_3 \leq 430$

$3x_1 + x_2 \leq 460$

$x_1 + 4x_3 \leq 420$

$x_1,\ x_2,\ x_3 \geq 0.$

(f) Use Big M method to solve the following LPP : 6

Maximize $Z = 2x_1 + x_2 + 3x_3$

subject to $x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1,\ x_2,\ x_3 \geq 0.$

(g) Solve the following LPP by two-phase method : 6

Minimize $Z = 4x_1 + x_2$

subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1,\ x_2 \geq 0.$

(h) (i) Show that the following LPP has no feasible solution :

Maximize $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1,\ x_2 \geq 0.$

(ii) Prove that the set of all feasible solutions of a linear programming problem is a convex set. 3+3

Group - C

3. Answer ***any five*** questions :

(a) Find the optimal solution of the following LPP by solving its dual : 6

Minimize $Z = 3x_1 + 2x_2$

subject to $7x_1 + 2x_2 \geq 30$

$5x_1 + 4x_2 \geq 20$

$2x_1 + 8x_2 \geq 16$

$x_1, x_2 \geq 0.$

(b) Solve the following transportation problem by Vogel's approximation method : 6

	D_1	D_2	D_3	D_4	Available
O_1	15	20	13	21	30
O_2	22	15	19	14	12
O_3	16	12	14	31	18
O_4	24	23	15	30	10
Demand	25	20	15	10	

(c) Solve the following travelling salesman problem : 6

	A	B	C	D	E
A	∞	7	6	8	4
B	7	∞	8	5	6
C	6	8	∞	9	7
D	8	5	9	∞	8
E	4	6	7	8	∞

(d) (i) State the maximin or minimax theorem.

(ii) Find the value of the following 2×2 game algebraically by using mixed strategies :

Player B

Player A	B_1	B_2
A_1	2	3
A_2	4	-1

2+4

(e) Solve the game problem using the dominance property : 6

Player B

Player A

0	2	0	1	1
4	2	0	2	1
4	3	1	3	2
4	3	4	–1	2

(f) Solve the following 2×4 game graphically : 6

Player B

Player A	B_1	B_2	B_3	B_4
A_1	6	5	2	3
A_2	1	2	6	3

(g) Solve the following game problem by using linear programming method : 6

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \end{array}\begin{array}{c} \text{Player B} \\ \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} 1 & 0 & -2 \\ 0 & 3 & -2 \end{pmatrix} \end{array}$$

(h) If X be any feasible solution to a primal problem and V be any feasible solution to its dual problem, then prove that $CX \leq b'V$, where C and b are respectively the cost vector and the requirement vector of the primal problem. 6
