Write the answers to each *Group* in a separate answer-book.

2025

MATHEMATICS — MDC

Paper : CC-1

(Calculus, Geometry and Vector Analysis)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

Group - A

(Calculus)

(Marks : 20)

1. Answer ***any four*** questions : 2×4

(a) Find $\lim_{x\to 0} \frac{x - \sin x}{x^3}$.

(b) $y = 2\cos x(\sin x - \cos x)$, show that $(y_{10})_0 = 2^{10}$.

(c) Evaluate $\int_0^{\pi/2} \sin^9 x\, dx$ using the reduction formula $I_n = \frac{n-1}{n} I_{n-2}$, where $I_n = \int_0^{\pi/2} \sin^n x\, dx$.

(d) Find $\frac{dy}{dx}$, if $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$.

(e) Find the range of x in which the function $2x^3 + 3x^2 - 36x + 2026$ is monotonically increasing.

(f) Find the length of the arc of the parabola $x^2 = 4y$ from the vertex to the point where $x = 2$.

(g) Find the area bounded by curve $y = x^2$ and the line $y = 2x$.

2. Answer ***any three*** questions : 4×3

(a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, |x| < 1$, show that $(1 - x^2)y_{n+2} - (2n+3)\,x\,y_{n+1} - (n+1)^2 y_n = 0$.

(b) Find a and b such that

$$\lim_{x\to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1.$$

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(c) Show that $x > \log(1+x) > x - \frac{x^2}{2}$, for $x > 0$.

(d) If $I_n = \int_0^{\frac{\pi}{2}} \cos^{n-2} x \sin nx \, dx$, show that $2(n-1)I_n = 1 + (n-2)I_{n-1}$.

(e) Find the perimeter of $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

(f) Find the volume of the solid obtained by revolving the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 1$, $x = 4$ about x-axis.

Group - B

(Geometry)

(Marks : 35)

3. Answer ***any two*** questions : 2½×2

(a) Find the angle of rotation of the axes about the origin which transforms the equation

$x^2 - y^2 = 4$ to $x'y' = 2$.

(b) Find the point on the conic $\frac{14}{r} = 3 - 8\cos\theta$ whose vectorial angle is $\frac{2\pi}{3}$.

(c) Find the equation of the sphere passing through origin and the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$.

(d) Show that the equation $2x^2 + 3y^2 - 8x + 6y - 12z + 11 = 0$ represents an elliptic paraboloid.

4. Answer ***any five*** questions : 6×5

(a) If the normal to the hyperbola $xy = c^2$ at the point $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve again at the point $\left(ct_2, \frac{c}{t_2}\right)$, then show that $t_1^3 t_2 + 1 = 0$.

(b) Show that the straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ touches the conic

$\frac{l}{r} = 1 + e\cos\theta$ if $(A - e)^2 + B^2 = 1$.

(c) If PSQ and $PS'R$ be two focal chords of an ellipse through the foci S and S', then show that

$$\frac{SP}{SQ} + \frac{S'P}{S'Q}$$

is independent of the position of P.

(d) Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to its canonical form and determine the nature of the conic.

(e) Find the equation of the sphere which passes through the origin and touches the sphere $x^2 + y^2 + z^2 = 56$ at the point $(2, -4, 6)$.

(f) Find the equation of the cone whose vertex is origin and which contains the curve

$$x^2 - y^2 + 4x = 0, \quad x + y + z = 3.$$

(g) Find the equation of the cylinder generated by straight lines parallel to

$$\frac{x}{1} = \frac{y}{5} = \frac{z}{-2},$$

the guiding curve being the conic $x = 0$, $y^2 = 6z$.

(h) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z.$$

(i) Find the nature of the surface represented by the equation $2x^2 + 5y^2 + 3z^2 - 4x + 20y - 6z - 5 = 0$.

Group - C
(Vector Analysis)
(Marks : 20)

5. Answer ***any four*** questions : 2×4

(a) For any two vectors $\vec{\alpha}$ and $\vec{\beta}$ show that

$$|\vec{\alpha} \times \vec{\beta}|^2 + |\vec{\alpha} \cdot \vec{\beta}|^2 = |\vec{\alpha}|^2 \, |\vec{\beta}|^2 .$$

(b) If a particle is acted on by constant forces $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + 5\hat{k}$ and is displaced from the point $(\hat{i} + 2\hat{j} - 3\hat{k})$ to $2\hat{i} - 3\hat{j} + 7\hat{k}$, then find the total work done by the forces.

(c) Three forces $2\hat{i} + 3\hat{j} - 5\hat{k}$, $-5\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - 2\hat{j} + 4\hat{k}$ act on a particle, find the resultant of the forces.

(d) Find the vector equation of a straight line passing through the points (2, 3, 4) and (–1, 3, –2).

(e) Find the volume of the tetrahedron whose vertices are (1, 1, 0), (1, 0, 1), (0, 1, 1) and (1, 1, 1).

(f) If $\vec{r} = \left(3t, 3t^2, 2t^3\right)$, find $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right]$.

(g) If $\vec{F}(t) = 3t^2\hat{i} + t\hat{j} - t^3\hat{k}$, then find $\int_1^2 \vec{F}(t) \times \frac{d^2\vec{F}(t)}{dt^2}\,dt$.

6. Answer ***any three*** questions :

(a) $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three non-coplanar vectors, then show that $\left(\vec{\alpha} - \vec{\beta}\right)\cdot\left(\vec{\beta} - \vec{\gamma}\right)\times\left(\vec{\gamma} - \vec{\alpha}\right) = -\left[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}\right]$. 4

(b) Find the shortest distance between the straight lines

$\vec{r} = \vec{\alpha} + t\vec{\beta}$ and $\vec{r} = \vec{\gamma} + s\vec{\delta}$, where $\vec{\alpha} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{\beta} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{\gamma} = \lambda\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{\delta} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. 4

(c) If $\vec{u} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$ and $\vec{v} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$, then show that

(i) $\frac{d}{dt}\left(\vec{u}.\vec{v}\right) = -6$ at $t = 1$

(ii) $\frac{d}{dt}\left(\vec{u} \times \vec{v}\right) = 7\hat{j} + 3\hat{k}$ at $t = 1$. 2+2

(d) If $\frac{d\vec{a}}{dt} = \vec{\omega} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{\omega} \times \vec{b}$,

show that $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right) = \vec{\omega} \times \left(\vec{a} \times \vec{b}\right)$. 4

(e) If $\vec{F} = xy\hat{i} - 2xy^2\hat{j} + zxy^3\hat{k}$ and $\vec{G} = 2x\hat{i} + y\hat{j} - zx^2\hat{k}$,

then find $\frac{\partial^2}{\partial x\,\partial y}\left(\vec{F}\times\vec{G}\right)$ at (1, 1, –1). 4

(f) If $\vec{r}(t) = \left(t - t^2\right)\hat{i} + 2t^3\hat{j} - 3\hat{k}$, find $\int_1^2 \vec{r}\,dt$. 4

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