2025

PHYSICS — HONOURS

Paper : DSCC-1

(Basic Physics - I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions, taking ***at least one*** question from ***each Group***.

1. Answer ***any five*** questions : 3×5

(a) Solve $y' = x - y$.

(b) Sketch the polar curves $\theta = \frac{\pi}{4}$ and $r = 2$ on the same graph. Express both the curves in terms of Cartesian coordinates x and y.

(c) If $\vec{\omega}$ is a constant vector and $\vec{v} = \vec{\omega} \times \vec{r}$, then show that $\vec{\nabla} \cdot \vec{v} = 0$.

(d) Show that the magnitude of the integral $\oint \vec{r} \times d\vec{\theta}$ over a circle of radius r, with center at the origin is $2\pi r$.

(e) Assuming that the planet Jupiter has periods of 4333 earth days approximately, find the mean distance of Jupiter from the Sun. Consider the distance of the Earth from the Sun to be 150×10^6 km.

(f) Two particles moving along x-axis collide head on. One was moving at 3.6 m/s in the positive x direction and the second was moving at 2.4 m/s in the opposite direction. After the collision, the second one of mass five times the first one continues moving in its initial direction of motion at 0.24 m/s. What is the final velocity of the first?

(g) Show that for a particle moving in a central force field, motion is confined to a plane.

(h) What is meant by an ideal fluid? Distinguish between streamline flow and turbulent flow in fluids.

Group - A

2. (a) Show that the differential $df = x^2 dy - (y^2 + xy)dx$ is not exact, but that $dg = (xy^2)^{-1} df$ is exact.

(b) Obtain the first three non-zero terms in the Taylor series of $\sin x$ about $x = 0$. Hence, obtain the series expansion of cosec x about $x = 0$ using binomial expansion upto $O(x^3)$.

(c) Show that the solution to the differential equation $x''(t) + 2x'(t) + x(t) = 0$ with initial conditions $x(0) = 0$ and $x'(0) = 1$ has its maximum value at $t = 1$. Here $'$ indicates derivative with respect to t.

3+5+4

3. (a) Find the directional derivative of $\phi(x, y, z) = 4xz^3 - 3x^2y^2z$ at $(1, -1, -1)$ in the direction $2\hat{i} - 3\hat{j} + 6\hat{k}$.

(b) (i) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{A} = \vec{\nabla}\phi$.

(ii) Using divergence theorem, evaluate $\iint_S \vec{A}\cdot\hat{n}\,ds$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

3+(3+3)+3

4. (a) (i) Write down the unit vectors $(\hat{r}, \hat{\theta})$ of the plane polar system in terms of the unit vectors $(\hat{i}, \hat{j})$ of the 2-dimensional Cartesian system.

(ii) Hence, verify that $(\hat{r}, \hat{\theta})$ form an orthogonal system.

(iii) Compute $\oint \hat{r}\,d\theta$ over a circle centered at the origin.

(b) For a given vector $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$, evaluate $\iint \vec{V}.\hat{n}\,ds$ on the closed surface of the cylinder (including the top and the bottom surfaces) of radius R and length L.

(the centre of the bottom circular disc is at the origin of the coordinate system and the axis of the cylinder is along the z-axis.)

(c) The divergence of a vector field vanishes at all points in space. What does this signify about the vector field?

(2+2+2)+4+2

Group - B

5. (a) A particle of mass m moves in the xy plane so that its position vector is $\vec{r} = \hat{i}\,a\cos\omega t + \hat{j}\,b\sin\omega t$ (where a, b and ω are positive constants with $a > b$). Show that the particle moves in an ellipse.

(b) Prove that the force acting on the particle is a centripetal force.

(c) Show that the force is also conservative.

(d) Find out the instantaneous power applied to the particle.

(e) Find the potential energy and kinetic energy of the particle and demonstrate the principle of conservation of energy.

2+2+2+2+(2+1+1)

6. (a) Define stable and unstable equilibrium.

(b) A particle of mass m moving in the potential $U(x, y, z) = \alpha x + \beta y^2 + \gamma z^3$, where α, β, γ are constants, has a speed v_0 when it passes through the origin.

(i) What will its speed be when it passes through the point (1, 1, 1)?

(ii) If $v = 0$ at (1, 1, 1), what is v_0?

(c) A particle moves in a potential field $U(x) = -ax + bx^2$ where $a > 0$ and $b > 0$ are constants.

(i) Sketch the potential and the force on the same graph.

(ii) Find the equilibrium position of the particle and the potential at that point. Also, state the nature of the equilibrium. 2+(2+2)+(3+2+1)

7. (a) A triangle has masses 1 kg, 2 kg and 4 kg located at its vertices $A(-1, 2, -2)$, $B(3, 2, -1)$ and $C(0, -1, 1)$ respectively. Find the coordinates of the center of mass.

(b) Prove that the total linear momentum of a system of particles about the center of mass is zero.

(c) Show that the total kinetic energy of a system of particles about any point equals the kinetic energy of the center of mass plus the kinetic energy of motion about the center of mass.

(d) Two particles having masses m_1 and m_2 move so that their relative velocity is $\vec{v}$ and the velocity of their center of mass is $\vec{v_c}$. If M and μ are the total mass and the reduced mass of the system respectively, prove that the total kinetic energy is $\frac{1}{2}Mv_c^2 + \frac{1}{2}\mu v^2$. 2+3+3+4

8. (a) (i) For a particle of mass m moving in (r, θ) plane, show that the radial and the transverse acceleration are given by

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\dot{\theta}^2 + 2\dot{r}\dot{\theta}$$

(ii) Setting $a_\theta = 0$ in a central force field, show that $mr^2\dot{\theta}$ is constant in time.

(iii) If the particle moves in an orbit $r = a\theta$, where $\theta = bt$ with constants a and b, determine whether the force field is central.

(b) By substituting $r = 1/u$, show that the differential equation for the path of the particle is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}.$$

(3+2+3)+4

9. (a) (i) Show that for a fluid of density ρ and velocity $\vec{v}$, moving in a gravitational field $\vec{g}$, the equation of motion is

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \cdot \vec{v} = -\frac{1}{\rho} \vec{\nabla} P + \vec{g},$$

where P is the pressure inside the fluid and the density ρ is constant.

(ii) Hence, show that for a fluid at rest in the absence of gravity, the pressure is the same at all points.

(iii) Obtain Bernoulli's theorem, from (i).

(b) An open tank of base area A contains water up to a height h. Water drains out through a hole of area $a \ll A$ in the base with speed v. Show that $v = \sqrt{2gh}$, where g is the acceleration due to gravity. (4+2+3)+3