2025

PHYSICS — HONOURS

Paper : DSCC-4

(Mathematical Physics - I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions from the rest.

1. Answer ***any five*** questions : 3×5

(a) Check the convergence of the series $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots\ldots\ldots+\frac{1}{\sqrt{n}}+\ldots\ldots\ \infty$.

(b) Expand
$$f(x) = \frac{1}{4} - x, \quad \text{when } 0 < x < \frac{1}{2}$$
$$= x - \frac{3}{4}, \quad \text{when } \frac{1}{2} < x < 1$$
in the Fourier Sine series.

(c) Consider the two-dimensional Laplace's equation
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$
Let $P = x + \lambda y$ and assume $u(x, y) = f(P)$ to be a solution of this equation. Show that $\lambda^2 + 1 = 0$.

(d) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(e) Evaluate $\int_{-1}^{+1} e^x\, \delta'(x)\,dx$ when the prime denotes differentiation with respect to x.

(f) Find the Fourier series for the periodic function $f(x) = \sin^4 x$.

(g) Let $Ae^{-|x-a|}$ be the probability density function for a random variable $x \in (-\infty, \infty)$. Find the mean $\langle x \rangle$.

(h) Find the root of the polynomial $f(x) = x^3 - x - 2$ using bisection method (3rd iteration), given the root lies between 1.5 and 1.75.

2. (a) What is meant by absolute convergence of an infinite series?

(b) Consider the alternating series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... \frac{(-1)^{n+1}}{n} + ...$$

Show that it's not absolutely convergent but converges.

(c) Consider the series, $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + ... + \frac{(-x)^n}{2^n} + ... \infty.$

(i) Find out the condition for which the series converges.

(ii) What is the interval of convergence?

(d) Assume u_i's and v_j's are non negative real numbers for all i, j. If the series $u_1^2 + u_2^2 + ...$ & $v_1^2 + v_2^2 + ...$ are both convergent, then show that $u_1v_1 + u_2v_2 + ...$ is also convergent. 2+4+3+3

3. (a) Prove that for $0 \le x \le \pi$ there are two expansions :

(i) $x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + ...\right)$

(ii) $x(\pi - x) = \frac{8}{\pi}\left(\frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + ...\right).$

(b) Use the above to find the following series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^3} = \pi^3/32.$ (4+4)+(2+2)

4. (a) A string, clamped at $x = 0$ and $x = 1$ is vibrating in a damping medium according to the equation :

$$\frac{\partial^2 u(x,t)}{\partial t^2} = v^2 \frac{\partial^2 u(x,t)}{\partial x^2} - k\frac{\partial u(x,t)}{\partial t}.$$

Solve the P.D.E. for the given initial condition of the string :

$$u(x,0) = f(x) \text{ and } \frac{\partial u(x,0)}{\partial t} = g(x).$$

(b) Consider the one-dimensional heat flow equation in a bar of length l, where the temperature distribution function $T(x, t)$ satisfies the heat equation :

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2}.$$

subjected to the boundary condition $T(0, t) = T(l, t) = 0$.

(c) Solve $\nabla^2 V = 0$ in a region $r \geq R$ where $V = V(r, \theta)$. Given boundary condition $V(R, \theta) = V_0 \cos\theta$ and $V(r, \theta) \to 0$ for $r \to \infty$. 6+3+3

5. (a) Evaluate the following integral :

$$\int_{\frac{1}{2}}^{\infty} [x] e^{-x}\, dx,$$

where the function $[x]$ is the greatest integer not exceeding x.

(b) Show that $\delta(ax) = \frac{\delta(x)}{|a|}$.

(c) Evaluate the integral :

$$\int_V \left(r^4 + r^2 \left(\vec{r}.\vec{c}\right) + c^4\right) \delta^3 \left(5\vec{r} - \vec{c}\right) dxdydz,$$

where $\vec{c} = 5\hat{i} + 3\hat{j} + 2\hat{k}$ and V is a sphere of radius 6 centered about origin. 4+4+4

6. (a) Assuming that the Fourier expansion of $f(x)$ is uniformly convergent over $(-\pi, \pi)$, show that

$$\frac{1}{\pi} \int_{-\pi}^{\pi} [f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} \left[a_n^2 + b_n^2\right].$$

(b) Gaussian distribution function is given as $f(x) = Ne^{-\alpha x^2}$,

when N and α are constants.

(i) Obtain the Fourier Transform of $f(x)$.

(ii) Is the transform also Gaussian?

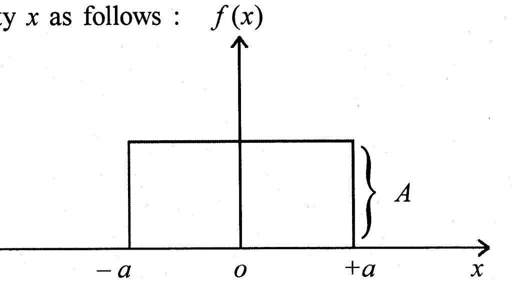
(iii) Show that the product of the widths of the function in x-space and its transformed space is constant.

(iv) Discuss the physical implication of this result for small α. 3+(4+1+2+2)

7. (a) Let $f(x)$ be the probability distribution of the quantity x as follows :

(i) Find the value of A for which the function is normalized.

(ii) Calculate the mean values of x, x^2 and $|x|$.



(b) The probability distribution function of the quantity x has the form $f(x) = A\, 4\pi x^2 e^{-x^2}$. Find the constant A.

(c) The overall percentage of failure in an examination is 20%. If 8 students appear in the examination, what is the probability that at least 7 students pass? (3+5)+2+2

8. (a) Define Beta function, $\beta(p, q)$.

(b) Show that $\beta(p, q) = \beta(q, p)$.

(c) Evaluate $\int_0^1 x^4 (1-x)^3\, dx$.

(d) Define the Gamma function $\Gamma(x)$.

Show that $\beta(x, y) = \dfrac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$. 3+3+3+(1+2)

9. (a) Construct the Lagrange interpolation polynomial for the data :

x	–1	1	4	7
$f(x)$	–2	0	63	342

Hence, interpolate at $x = 5$.

(b) Given the function $f(x) = x(1+x)^{-\frac{1}{2}}$, find its root using Newton-Raphson method up to four iterations, by choosing the initialization between [0.5, 1).

(c) Consider the initial-value problem :

$$\frac{dy}{dx} = y - x,\ y(0) = \frac{1}{2}.$$

Use Euler's method with step size $h = 0.1$, to obtain an approximation to $y(0.5)$. (3+2)+3+4