2026

MATHEMATICS — HONOURS

Paper : DSCC-14

(Multivariable Calculus II and Application of Calculus)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and Symbols have their usual meaning.

Group - A

(Multivariable Calculus II)

1. Answer ***any five*** questions : 3×5

(a) Prove that the function $\tilde{f}:\mathbb{R}^n \to \mathbb{R}$ defined by $\tilde{f}(\tilde{x}) = \|\tilde{x}\|$ is continuous.

(b) Write the Jacobian matrix for the function $\tilde{f}:\mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$f(x, y, z) = (x^2 + yz,\ \tan^{-1}(x+z),\ z - ye^x).$$

(c) State the inverse function theorem for a function $\tilde{f}:\mathbb{R}^n \to \mathbb{R}^n$.

(d) Find the derivative of the function $f \equiv f(x, y) = 2x^2 + y^2$ at the point (5, 5) along a line whose direction cosines are $\frac{4}{5}$ and $\frac{3}{5}$.

(e) For what value of the constant a will the vector field given by

$$\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$$

be always irrotational?

(f) State Green's theorem in plane.

(g) Evaluate $\int_C \left(y^2 dx + xdy\right)$, where C is the arc of parabola $x = 4 - y^2$ from (–5, –3) to (0, 2).

(h) Show that the vector field given by $(y + \sin z)\hat{i} + x\hat{j} + (x\cos z)\hat{k}$ is conservative.

2. Answer ***any five*** questions :

(a) (i) Suppose E is an open set in $\mathbb{R}^n$ and $\tilde{f}: E \to \mathbb{R}^m$. Let $\tilde{x} \in E$. When is $\tilde{f}$ said to be differentiable at $\tilde{x}$?

(ii) If $\tilde{f}$ is differentiable at $\tilde{x}$, prove that there exists a unique linear transformation T of $\mathbb{R}^n$ into $\mathbb{R}^m$ such that

$$\lim_{\|\tilde{h}\| \to 0} \frac{\| \tilde{f}(\tilde{x}+\tilde{h}) - \tilde{f}(\tilde{x}) - T(\tilde{h}) \|}{\| \tilde{h} \|} = 0.$$

(Symbols have their usual meaning). 1+5

(b) Suppose E is an open set in $\mathbb{R}^n$ and $\tilde{f}: E \to \mathbb{R}^m$. Let $\tilde{f} = (f^1, f^2, \cdots, f^m)$ with $f^i : E \to \mathbb{R}$ for each $i = 1, 2, ..., m$. If $\tilde{f}$ is differentiable at $\tilde{x} \in D$, prove that each of the functions f^i is differentiable at $\tilde{x}$ and the derivative $\tilde{f}'(\tilde{x})$ can be represented by the Jacobian matrix

$$\tilde{f}'(\tilde{x}) = \begin{pmatrix} \dfrac{\partial f^1}{\partial x_1} & \dfrac{\partial f^1}{\partial x_2} & \cdots & \dfrac{\partial f^1}{\partial x_n} \\ \dfrac{\partial f^2}{\partial x_1} & \dfrac{\partial f^2}{\partial x_2} & \cdots & \dfrac{\partial f^2}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial f^m}{\partial x_1} & \dfrac{\partial f^m}{\partial x_2} & \cdots & \dfrac{\partial f^m}{\partial x_n} \end{pmatrix},$$

where the partial derivatives are evaluated at $\tilde{x}$. 6

(c) Prove that $\vec{\nabla}.\left(\vec{F} \times \vec{G}\right) = \vec{G}.\left(\vec{\nabla} \times \vec{F}\right) - \vec{F}.\left(\vec{\nabla} \times \vec{G}\right)$. 6

(d) If $\vec{F} = \left(5xy - 6x^2\right)\hat{i} + (2y - 4x)\hat{j}$, evaluate the line integral $\int_C \vec{F}.d\vec{r}$ along the curve C in the xy-plane, $y = x^3$ from the point (1, 1) to (2, 8). 6

(e) Find the constants a and b such that the vector field

$$\vec{F} = (x + 2y + 4z)\hat{i} + (2ax + by - z)\hat{j} + (4x - y + 2z)\hat{k}$$

is both solenoidal and irrotational. 6

(f) Using Green's theorem in the xy-plane, evaluate $\oint_C xe^{-2x}dx + (x^4 + 2x^2y^2)dy$, where C is the boundary of the semi-annular region $R = \left\{(x, y) \mid 1 \le x^2 + y^2 \le 4,\ y > x\right\}$. 6

(g) Use Stokes' theorem to find the line integral $\int_C \left(x^2y^3dx+dy+zdz\right)$, where C is the circle $x^2+y^2=a^2,\ z=0.$ 6

(h) Using Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$, where $\vec{F}=x\hat{i}+y\hat{j}+z^2\hat{k}$, and S is the closed surface bounded by the cone $x^2+y^2=z^2$ and the plane $z=1$. 6

Group - B

(Application of Calculus)

3. Answer ***any six*** questions : 5×6

(a) Show that the portion of the tangent at any point on the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ intercepted between the coordinate axes is of constant length.

(b) Show that the pedal equation of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with respect to focus as pole is $\frac{b^2}{p^2}=\frac{2a}{r}-1$.

(c) Find the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$.

(d) Show that at any point of the curve $x^{m+n}=k^{m-n}y^{2n}$, the m-th power of subtangent varies as the n-th power of the subnormal.

(e) The tangents at two points P, Q on the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, are at right angles. Show that if ρ_1, ρ_2 are the radii of curvature at these points then $\rho_1^2+\rho_2^2=16a^2$.

(f) Show that the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=\left(a^2-b^2\right)^{\frac{2}{3}}$.

(g) Show that the envelope of circle whose centres lie on the rectangular hyperbola $xy=c^2$ and which pass through its centre is $(x^2+y^2)^2=16c^2xy$.

(h) Find the range values of x for which the curve $y=x^4-16x^3+42x^2+12x+1$ is concave upwards or downwards. Also determine the points of inflexion.

(i) Determine the nature of the double points, if any of the curve $x^3-y^3-7x^2+4y+15x-13=0$.