2026

MATHEMATICS — MDC

Paper : CC-7

(Mathematical Methods)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

Group - A

Answer ***any five*** questions.

1. When is a sequence of functions $\{f_n\}_n$, defined on an interval $I \subseteq \mathbb{R}$ said to be uniformly convergent on I? Show that the sequence $\{f_n\}_n$, where $f_n(x) = \dfrac{x}{nx+1}$, $\forall x \in [0,1], n \in \mathbb{N}$ converges uniformly to 0 on [0, 1]. 1+4

2. Find the sum function of the series of function $\sum_{n=1}^{\infty} f_n$,

$$\text{where } f_n(x) = \frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}, \forall x \in [0,1], n \in \mathbb{N}$$

and show that at $x = 0$

$$\frac{d}{dx}\left(\sum_{n=1}^{\infty} f_n(x)\right) \neq \sum_{n=1}^{\infty} \frac{d}{dx}(f_n(x)).$$ 2+3

3. Prove that the series of functions $1 + x + x^2 + \ldots$ is convergent on $0 \le x < 1$, but the convergence is not uniform on [0, 1). 2+3

4. (a) State Weierstrass's M-test.

   (b) Using Weierstrass's M-test, prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\left(n^3 + n^4x^2\right)}$ is uniformly convergent for all real x. 2+3

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5. If $f_n(x)=\dfrac{\sin nx}{\sqrt{n}}, \forall x\in[0,1], n\in\mathbb{N}$, show that the sequence $\{f_n\}_n$ converges pointwise on [0, 1] to a differentiable function f. Does the sequence $\{f_n(0)\}_n$ converge? Justify 3+2

6. Prove that if a power series $a_0+a_1x+a_2x^2+\ldots.$ converges for $x=x_1$, then the series converges absolutely for all real x satisfying $|x|<|x_1|$. 5

7. Let $D\subseteq\mathbb{R}$ and $f_n : D\to\mathbb{R}$ be continuous on D for each $n\in\mathbb{N}$. Illustrate with the help of an example that the uniform convergence of the sequence $\{f_n\}_n$ on D is a sufficient but not a necessary condition for continuity of the limit function f on D. 5

8. Assuming the power series expansion $(1+x)^{-1}=1-x+x^2-x^3+\ldots$, $|x|<1$, show that $\log(1+x)=x-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\ldots, -1<x\le 1$. By using Abel's theorem, deduce $\log 2=1-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\ldots$ 5

9. Find the radius of convergence of the power series :

(a) $\dfrac{1}{3}-x+\dfrac{x^2}{3^2}-x^3+\dfrac{x^4}{3^4}-x^5+\ldots$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n+1}(x+1)^n$. 2+3

Group - B

Answer ***any seven*** questions.

10. If the normal to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ makes an angle ϕ with the axis of X, show that its equation is $y\cos\phi - x\sin\phi = a\cos 2\phi$. 5

11. Show that the radius of curvature at any point on the cardiode $r=a(1+\cos\theta)$ is $\dfrac{2}{3}\sqrt{2ar}$. 5

12. Obtain the pedal equation of the curve $x^2-y^2=a^2$ with respect to the centre. 5

13. Find the rectilinear asymptotes, if any of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$. 5

14. Obtain the envelope of the circle drawn upon the radii vectors of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ as diameter. 5

15. Find the position and nature of double point (if any) of the curve $y^2-x(x+a)^2=0$. 5

16. Using Lagrange's method of Undetermined Multipliers, find the minimum value of $(x^2 + y^2 + z^2)$ subject to the condition $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$. 5

17. Examine the extreme values, if $f(x) = x^5 - 5x^4 + 5x^3 + 12$. 5

18. Examine, whether the origin is a single cusp of first species of cissoid $y^2 = \frac{x^3}{2a-x}$. 5

19. Find the range of values of x in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is

(a) concave and (b) convex w.r.t. the x-axis; and (c) find the points of inflexion. 2+2+1

20. Examine for extreme value (if any) of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. 5

Group - C

Answer ***any three*** questions.

21. Find the Fourier series expansion for the function $f(x) = |x|$ in $[-\pi, \pi]$ and hence find the value of $1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$ 5

22. (a) State the Dirichlet's conditions for convergence of a Fourier series.

(b) Verify if the following statement holds with proper justification :

The function $f(x)=\frac{1}{(x-2)}, (0 \le x \le 3, x \ne 2)$ cannot be expanded as a Fourier series. 3+2

23. Find the Fourier cosine series for $f(x)=e^x,\ 0<x<\pi$. 5

24. If the Laplace transform of $F(t)$ be $f(p)$, then show that the Laplace transform of $e^{at}F(t)$ is $f(p-a)$.

Use this to show that $L\left\{e^t \sin^2 t\right\} = \frac{1}{2(p-1)} - \frac{p-1}{2\left(p^2-2p+5\right)}$. 5

25. Using Laplace transformation, solve : $\left(D^2+1\right)y = \sin 3t$, given that $y = 0$ and $Dy = 0$ when $t = 0$; where $D \equiv \frac{d}{dt}$. 5

26. Using Laplace transformation, solve : $y''+3y'+2y=e^{-t}$, given that $y(0)=y'(0)=0$. 5

27. Find the value of $L^{-1}\left\{\frac{4p-3}{p^2+4}\right\}$, where $L\{F(t)\} = f(p)$. 5
