2026

MATHEMATICS — MINOR

Paper : MN-4

(Mechanics)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any six*** questions : 2×6

(a) Two forces whose magnitude are P and $\sqrt{2}P$ gms-wt act on a particle in directions inclined at an angle of 135° to each other. Find the magnitude of the resultant.

(b) State the Varignon's theorem.

(c) State the principle of conservation of linear momentum.

(d) If a wheel makes 200 revolutions per hour, then find its angular velocity.

(e) The speed v of a point moving along the x-axis is given by $v^2 = 16 - x^2$. Prove that the motion is simple harmonic and find its period.

(f) The law of motion of a particle in a straight line is $x = \frac{1}{2}vt$. Show that the acceleration is constant, where x is the distance traversed by the particle at time t with velocity v.

(g) If the radial velocity is proportional to the transverse velocity, find the path in polar coordinate.

(h) Define Impulse and Impulsive Force.

(i) State Keplar's laws on planetary motion.

(j) For a rectilinear motion of a particle if an impulse I changes its velocity from u to v and E is the change of kinetic energy, show that $E = I\left(\frac{u+v}{2}\right)$.

Group - B

2. Answer ***any seven*** questions : 6×7

(a) Two forces P and Q acting on a particle at an angle α have a resultant $(2k+1)\sqrt{P^2+Q^2}$. When they act at an angle $90° - \alpha$, the resultant becomes $(2k-1)\sqrt{P^2+Q^2}$; prove that $\tan\alpha = \frac{k-1}{k+1}$.

(b) A system of coplanar forces has the total moment H, $2H$ and $3H$ about the points (0, 0), (0, 1) and (2, 4) respectively. Find the component of their resultant parallel to the coordinate axes and the equation to its line of action.

(c) A particle of mass m moves in a straight line under an acceleration n^2x towards a point O on the line, where x is the distance from O. Show that if $x = a$ and $\dot{x} = u$ when $t = 0$, then at time t, $x = a\cos nt + \frac{u}{n}\sin nt$.

(d) A particle falls vertically from rest in a medium whose resistance varies as the velocity. Find the velocity and displacement at any time t.

(e) A particle moves from rest in a straight line under an attractive force $\mu \times (\text{distance})^{-2}$ per unit mass to a fixed point on the line. Show that, if the initial distance from the centre of force be $2a$, then the distance of the particle will be 'a' after a time $\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}$.

(f) A particle of mass 'm' is acted on by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$, μ being a constant, towards the origin. If it starts from rest at a distance 'a' from the origin, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$.

(g) A particle is moving as a projectile under gravity. To show that the sum of the kinetic and potential energies at any point of its trajectory is constant.

(h) A particle moving in a straight line is acted upon by a force which works at a constant rate and changes its velocity from u to v in passing over a distance x. Prove that the time taken is

$$\frac{3(u+v)x}{2(u^2+uv+v^2)}.$$

(i) An engine working at a constant rate H draws a load M against a resistance R. Show that the maximum speed is $\frac{H}{R}$ and the time taken to attain half this speed is $\frac{MH}{R^2}\left[\log 2 - \frac{1}{2}\right]$.

(j) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun is $\sqrt{\dfrac{2m^2gh}{M(m+M)}}$.

(k) Two perfectly inelastic bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy due to impact is $\dfrac{1}{2}\cdot\dfrac{m_1m_2}{m_1+m_2}(u_1-u_2)^2$.

Group - C

3. Answer ***any three*** questions : 7×3

(a) A particle moves in a plane such that its accelerations parallel to x and y axis are $k^2a \sin kt$ and $k^2a \cos kt$ respectively with the initial conditions $x=0, y=-a, \dfrac{dy}{dt}=0, \dfrac{dx}{dt}=-ka$ when $t=0$. Show that the equation of the path is $x^2+y^2=a^2$.

(b) Find the expression for the radial and the cross radial components of velocities and accelerations of a particle moving in a plane.

(c) Establish the formula $\dfrac{d^2u}{d\theta^2}+u=\dfrac{F}{h^2u^2}$ for the motion of a particle describing a central orbit under an attractive force F per unit mass, the symbols having usual meaning.

(d) A particle moves with a central acceleration $\mu \div (\text{distance})^2$; it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1}\left[\frac{\mu}{VR\left(V^2-\frac{2\mu}{R}\right)^{\frac{1}{2}}}\right].$$

(e) A particle describes the path $r^2=a^2\cos 2\theta$ under a force which is always directed towards the pole. Find the law of force.

(f) A particle moves under the central force μu^3 per unit mass, it is projected from an apse at a distance a with velocity $\dfrac{2}{a}\sqrt{\dfrac{\mu}{3}}$; show that the path is $r\cos\dfrac{\theta}{2}=a$.