

2026

ECONOMICS — HONOURS

Paper : DSCC-5

(Mathematical Economics - I)

Full Marks : 75

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Group - A

1. Answer **any ten** questions : 2×10

- (a) Define the domain and range of a function. Find the domain of the real-valued function

$$f(x) = [1/(x - 3)].$$

- (b) If a production function is given by $Q = AK^\alpha L^\beta$, state the mathematical condition for it to exhibit diminishing marginal returns to labour (L).
- (c) State the Hawkins-Simon conditions for the viability of a Leontief Static Open Model.
- (d) Show that a monotonic transformation of a concave utility function may not necessarily be concave, but it preserves quasi-concavity.
- (e) Given the total cost function $TC = 3Q^2 + 5Q + 12$, find the output level Q where the elasticity of total cost with respect to output is equal to 1.
- (f) Check whether the following matrix is non-singular. What is its rank?

$$A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$$

- (g) State L'Hospital's Rule for evaluating limits.
- (h) Define a homothetic function. How does it relate to a homogeneous function?
- (i) Write down the economic interpretation of the Lagrange multiplier (λ) in a standard utility maximization problem under a budget constraint.
- (j) Define the indirect utility function. How does it differ fundamentally from a direct utility function?
- (k) State Shephard's Lemma using mathematical notation.
- (l) Formulate the mathematical dual of the following primal Linear Programming Problem :
Minimize $C = 6x_1 + 4x_2$, subject to $2x_1 + x_2 \geq 10$, $x_1 + 3x_2 \geq 15$, and $x_1, x_2 \geq 0$.
- (m) Distinguish between slack and surplus variables in Linear Programming.

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- (n) State the relationship between the marginal utility function and the total utility function. If total utility is given by $U(x) = 10x - 0.5x^2$, find the marginal utility function $[MU_x]$.
- (o) If a firm faces a competitive market price P , a specific tax t per unit of output, and a total cost function $C(Q)$, write down the first-order condition for profit maximization.

Group - B

Answer *any five* questions.

2. Given the demand function $Q = 100 - 2P - 0.5P^2$, find the price elasticity of demand when $P = 10$. Is the demand elastic or inelastic at this price? 5
3. Prove that for a linearly homogeneous production function $Q = f(K, L)$, the total product can be completely exhausted by paying each factor its marginal product (Euler's Theorem). 5
4. Find the Eigen values and the corresponding Eigen vectors of the matrix :

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \quad 5$$

5. A competitive firm faces a market price $P = 9$, and the total cost function

$$TC = \frac{1}{3}Q^3 - 5Q^2 + 30Q + 10$$

- (a) Find the profit-maximizing output level.
- (b) Check the second-order sufficient condition to confirm it is indeed a maximum. 2+3
6. For a two-variable function $z = f(x, y)$, state the sufficient conditions for :
- (a) A local maximum using the Hessian determinant.
- (b) A local minimum using the Hessian determinant. 2½+2½
7. Explain algebraically how the introduction of an *ad valorem* tax at rate r alters the profit-maximizing equilibrium choice of an individual firm under perfect competition compared to a lump-sum tax. 5
8. Consider the utility function $U(x, y) = xy$. Find the slope and the curvature of the indifference curve at any point (x, y) . 5
9. A small firm produces two types of handcrafted chairs : standard (x_1) and deluxe (x_2). Each standard chair requires 2 hours of carpentry and 1 hour of finishing. Each deluxe chair requires 1 hour of carpentry and 3 hours of finishing. The available daily capacities are 10 hours for carpentry and 15 hours for finishing. The profit contribution is ₹ 400 for a standard chair and ₹ 600 for a deluxe chair. Formulate this setup as a classic profit-maximizing Linear Programming Problem. 5

Group - CAnswer *any three* questions.

10. A consumer faces the utility function $U(x_1, x_2) = x_1x_2$. The prices of the goods are p_1 and p_2 respectively and the consumer's income is M .
- Set up the Lagrangian optimization problem to maximize utility subject to the budget constraint.
 - Derive the Marshallian demand functions for both goods.
 - Derive the indirect utility function and verify Roy's Identity for good 1. 2+5+3
11. Consider a firm with the production function $Q = L^{1/2}K^{1/2}$. The wage rate is w and the rental rate of capital is r .
- Set up the cost-minimization problem to produce a target output Q^* .
 - Find the input demand functions for labour (L^*) and capital (K^*), and check second-order conditions.
 - Construct the total cost function $C(w, r, Q^*)$ and verify fields of the Envelope Theorem via Shephard's Lemma for labour. 2+5+3
12. An economy consists of two sectors, Agriculture (1) and Manufacturing (2). The structural matrix (A) and the final demand vector (d) are given by :

$$A = \begin{pmatrix} 0.2 & 0.3 \\ 0.4 & 0.1 \end{pmatrix}, \quad d = \begin{pmatrix} 120 \\ 180 \end{pmatrix}$$

- Check if the system satisfies the Hawkins-Simon viability conditions.
 - Calculate the gross output levels (X_1 and X_2) required to sustain this final demand using Cramer's rule or matrix inversion. 5+5
13. A consumer has the utility function $U(x, y) = x + 2y$. The market prices are $p_x = 2$, and $p_y = 2$, and the consumer's total budget is $M = 20$.
- Set up the optimization problem maximizing utility subject to the budget constraint and non-negativity constraints using Kuhn-Tucker conditions.
 - Solve for the optimal consumption bundle (x^*, y^*) using the Kuhn-Tucker conditions. 5+5
14. Consider the following Linear Programming Problem (LPP) representing a firm's output choice :
- Maximize $Z = 4x_1 + 3x_2$, subject to : $2x_1 + x_2 \leq 10$, $x_1 + 2x_2 \leq 12$, and $(x_1, x_2) \geq 0$.
- Find the optimal values of x_1, x_2 and maximum profit Z using the graphical method.
 - Formulate the dual of this primal problem. 5+5