

2026

## MATHEMATICS — HONOURS

Paper : DSCC-6

(Mechanics - I)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Symbols used in the question have their usual meanings.

Group – A

(Marks : 12)

1. Answer **any six** questions :

2×6

- (a) Find the angle between two forces of magnitudes  $P$  and  $\sqrt{3}P$ , when their resultant is of magnitude  $P$ .
- (b) State Varignon's Theorem.
- (c) If time  $t$  be regarded as a function of velocity  $v$ , prove that the rate of decrease of acceleration  $f$  is  $f^3 \frac{d^2t}{dv^2}$ .
- (d) Find the time period of the simple harmonic motion  $x = \cos\left(\frac{3\pi t}{4}\right)$ .
- (e) Two bodies of masses  $m$  and  $4m$  are moving in the same direction with equal kinetic energies. Find the ratio of their momentum.
- (f) A particle describes the curve  $r = ae^{\theta}$  with constant angular velocity  $\omega$ . Show that the transverse radial acceleration varies as its distance from the pole, where  $a$  is a constant.
- (g) A particle describes a curve  $s = c \tan \psi$  with uniform speed  $v$ . Find the normal component of acceleration. (Symbols have their usual meaning)
- (h) If the greatest height attained by a particle is one-fourth of its range on the horizontal plane, find the angle of projection.

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(6914)

- (i) A particle describes an ellipse under a force  $\mu/(\text{distance})^2$  towards the focus. It was projected with velocity  $V$  from a point distance  $r$  from the centre of force. Show that its periodic time is

$$\frac{2\pi}{\mu} \left[ \frac{2}{r} - \frac{V^2}{\mu} \right]^{-\frac{3}{2}}, \text{ assuming the expression of velocity } v \text{ for elliptic orbit as } v^2 = \mu \left( \frac{2}{r} - \frac{1}{a} \right).$$

- (j) Show that in a central orbit, the angular momentum is always conserved.

### Group – B

(Marks : 42)

2. Answer **any seven** questions :

6×7

- (a) A system of coplanar forces has the total moments  $H$ ,  $2H$  and  $3H$  about the points  $(0, 0)$ ,  $(0, 1)$  and  $(2, 4)$  respectively. Find the magnitude and the equation of the line of action of the resultant.
- (b) Three forces  $P$ ,  $Q$ ,  $R$  acting at the vertices  $A$ ,  $B$ ,  $C$  respectively of a triangle, each perpendicular to opposite side, keep it in equilibrium. Prove that  $P : Q : R = a : b : c$ .
- (c) A particle moves from rest in a straight line under an attractive force  $\pi \times (\text{distance})^{-2}$  per unit mass to a fixed point on the line. Show that if the initial distance from the centre of force be  $2a$ ,

then the distance will be  $a$  after a time  $\left(\frac{\pi}{2} + 1\right) \left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}$ .

- (d) A particle is projected upwards with a velocity  $U$  in a medium whose resistance varies as the square of the velocity. Prove that it will return to the point of projection with velocity  $v = \frac{UV}{\sqrt{U^2 + V^2}}$ ,

where  $V$  is the terminal velocity.

- (e) A particle of mass  $m$  moves on a straight line under an attractive force  $mn^2x$  towards a point  $O$  on the line, where  $x$  is the distance from  $O$ . If  $x = a$  and  $\frac{dx}{dt} = u$  when  $t = 0$ , then show that the

amplitude of the motion is  $\sqrt{a^2 + \frac{u^2}{n^2}}$ .

- (f) One end of an elastic string is fixed and to the other end is fastened a particle heavy enough to stretch the string to double its natural length  $a$ . The string is drawn vertically downwards till it is four times its natural length and then let go. Show that the particle returns to this string point in

time  $\left(2\sqrt{3} + \frac{4\pi}{3}\right) \sqrt{\frac{a}{g}}$ , where modulus of elasticity is equal to the weight of the particle.

(3)

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- (g) An engine is pulling a train and works at a constant power doing  $H$  units of work per second. If  $M$  be the mass of the whole train and  $F$  be the resistance (supposed constant), then prove that the time in seconds of generating velocity  $V$  from rest is  $\frac{MH}{F^2} \log \frac{H}{H-FV} - \frac{MV}{F}$ .
- (h) A body of mass  $(m_1+m_2)$  splits into two parts of masses  $m_1$  and  $m_2$  by an internal explosion which generates kinetic energy  $E$ . If after explosion the parts move in the same line as before, show that their relative speed is  $\sqrt{\frac{2(m_1+m_2)E}{m_1m_2}}$ .
- (i) A ball is thrown from a point in a smooth horizontal ground with a velocity  $V$  at an angle  $\alpha$  to the horizon. Assuming  $e$  to be coefficient of elasticity of the ball with the ground, show that the total time for which the ball rebound on the ground is  $\frac{2V \sin \alpha}{g(1-e)}$  and that its distance from the starting points where it ceases to rebound is  $\frac{V^2 \sin 2\alpha}{g(1-e)}$ .
- (j) A particle, moving in a straight line, is acted upon by a force which works at a constant rate and changes its velocity from  $u$  to  $v$  in passing over a distance  $x$ . Prove that the time taken is  $\frac{3(u+v)x}{2(u^2+uv+v^2)}$ .
- (k) A particle of mass  $m$  is moving as a projectile under the action of gravity. Show that the sum of the kinetic and potential energies at any point of its path is constant.

**Group – C****(Marks : 21)****3. Answer any three questions :****7×3**

- (a) Find the tangential and normal components of velocity and acceleration for a particle moving in a plane.
- (b) A particle moving in a plane under the central acceleration  $P$ , has an acceleration  $T$  perpendicular to  $P$ . Show that in usual notations the differential equation of the path is  $\frac{d^2u}{d\theta^2} + u = \frac{P - T \frac{du}{d\theta}}{h^2 u^2}$ .
- (c) A particle moves in one plane under a force which is always perpendicular and towards a fixed straight line on the plane, its magnitude being  $\mu / (\text{distance from the line})^2$ . If initially it be at a distance  $2a$  from the line and be projected with a velocity  $\sqrt{\frac{\mu}{a}}$  parallel to the line, prove that the path traced out by it is a cycloid.

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- (d) A particle, subject to a central force per unit mass, equal to  $\mu[2(a^2 + b^2)u^5 - 3a^2b^2u^7]$ ,

is projected at a distance  $a$  with a velocity  $\frac{\sqrt{\mu}}{a}$  in a direction right angle to the initial distance.

Find the path.

- (e) A particle moves in a plane curve such that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Then prove that the equation of the curve is  $as + b = ce^\psi$ .

- (f) A particle moves with a central acceleration  $\frac{\mu}{(\text{distance})^2}$ ; it is projected with velocity  $V$  at a distance  $R$ . Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1} \left( \frac{\mu}{VR \left( V^2 - \frac{2\mu}{R} \right)^{1/2}} \right).$$


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