

2026

## MATHEMATICS — HONOURS

Paper : DSCC-7

(Multivariate Calculus - I and Partial Differential Equations - I)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**All symbols have their usual meanings.*

Group – A

[Multivariate Calculus – I]

(Marks : 60)

1. Answer **any five** questions :

2×5

(a) Examine the continuity of the function  $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

at (0, 0).

(b) Find the directional derivative of  $\phi = x^2yz + 4xz^2$  at the point (1, -2, -1) in the direction of the vector  $2\hat{i} - \hat{j} - 2\hat{k}$ .

(c) Evaluate  $\iint_S (x + y) dS$  over the region  $S$  bounded by  $y = x^2$  and  $y^2 = x$ .

(d) Find the value of the integral  $\int_0^1 \int_y^{1-x} \int_0^{1-x-y} x^2 dz dx dy$ .

(e) Verify whether the set  $S = \{(x, y) \in \mathbb{R}^2 : 1 < x < 2, 1 < y < 2\}$  is an open set in  $\mathbb{R}^2$  or not.

(f) If  $x = u^3v^3$ ,  $y = u^3 + v^3$ , find the Jacobian  $\frac{\partial(x, y)}{\partial(u, v)}$ .

(g) Find the total differential of  $z = x^2y + y^3$ .

(h) Verify Euler's Theorem for the homogeneous function  $u = x^2 + 2xy + y^2$ .

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2. Answer **any five** questions :

10×5

- (a) (i) State and prove the sufficient condition of differentiability.  
 (ii) Using sufficient condition for differentiability, show that the following function is differentiable at  $(0, 0)$ .

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{when } x^2 + y^2 \neq 0, \\ 0, & \text{when } x^2 + y^2 = 0. \end{cases} \quad (2+4)+4$$

- (b) (i) Let  $z$  be a differentiable function of  $x$  and  $y$  and let

$$x = r \cos \theta, \quad y = r \sin \theta.$$

Prove that

$$\frac{\partial^2 z}{\partial r^2} + \frac{1}{r} \frac{\partial z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 z}{\partial \theta^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}.$$

- (ii) Let  $f(x, y) = \begin{cases} xy^2, & y > 0 \\ -xy^2, & y < 0 \end{cases}$ . Show that  $f_{xy}(0, 0) = f_{yx}(0, 0)$ . 5+5

- (c) (i) Evaluate  $\iiint z^2 dx dy dz$  extended over the hemisphere  $z \geq 0, x^2 + y^2 + z^2 \leq a^2$ .

- (ii) Evaluate  $\iint_E \left( 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right) dx dy$ , where  $E$  is the positive quadrant of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . 5+5

- (d) (i) State the Existence theorem of Implicit Function. Verify the Implicit Function Theorem for the equation

$$F(x, y) = x^2 + y^2 - 1 = 0$$

at the point  $(0, 1)$ . Hence obtain the implicit function  $y = f(x)$  in a neighbourhood of  $(0, 1)$ .

- (ii) Using Lagrange's Multiplier Method, find the shortest distance between  $(-1, 4)$  and the straight line  $12x - 5y + 71 = 0$ . (2+2+1)+5
- (e) (i) Find the surface area of that part of the sphere  $x^2 + y^2 + z^2 = a^2$  which lies inside the cylinder  $x^2 + y^2 = ax$ .
- (ii) Determine the volume of the solid bounded by the cylinder  $x^2 + y^2 = 2ax$  and the paraboloid  $y^2 + z^2 = 4ax$ . 5+5
- (f) (i) Show that the function  $f(x, y) = \sqrt{|xy|}$  is not differentiable at  $(0, 0)$ .

- (ii) Using the transformation  $x + y = u$ ,  $y = uv$ , show that  $\int_0^1 dx \int_0^{1-x} e^{x+y} dy$  becomes

$\iint_E e^{x+y} dx dy$ , where  $E$  is the triangle bounded by  $x = 0$ ,  $y = 0$ ,  $x + y = 1$  and whose value is

$$\frac{1}{2}(e-1). \quad 5+5$$

- (g) (i) Expand the function  $f(x, y) = -x^2 + 2xy + 3y^2 - 6x - 2y - 4$  by Taylor's Formula in the neighbourhood of the point  $(-2, 1)$ .  
 (ii) If  $f(x, y)$  is continuously differentiable in domain  $\mathbb{R}$ , then using Mean Value Theorem prove that  $f(x, y)$  is uniformly continuous in  $\mathbb{R}$ . 5+5
- (h) (i) Let  $u$  be a homogeneous function of  $x$  and  $y$  of degree  $n$ . Then

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

- (ii) Examine the maxima and minima of the function  $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ . 5+5

### Group – B

#### [Partial Differential Equations – I]

(Marks : 15)

3. Answer **any three** questions :

5×3

- (a) Form a partial differential equation by eliminating the function  $\phi$  from  $z = e^{ax+by}\phi(ax - by)$ .  
 (b) Solve the partial differential equation  $z(x+y)\frac{\partial z}{\partial x} + z(x-y)\frac{\partial z}{\partial y} = x^2 + y^2$  by Lagrange's Method.  
 (c) Find the complete integral of  $(p^2 + q^2)y - qz = 0$  by Charpit's method, where

$$p \equiv \frac{\partial z}{\partial x}, \quad q \equiv \frac{\partial z}{\partial y}.$$

- (d) Solve the Cauchy problem for the PDE :  $xu_x + yu_y = 2u$  subject to the initial condition  $u(x, 1) = x^3$ .  
 (e) Find the integral surface of the PDE :  $p + zq = z^2$  which passes through the initial curve  $x = 0$ ,  $z = y^2$ .