

2026

MATHEMATICS — MDC

Paper : CC-4

(Mechanics)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group - A

1. Answer *any six* questions :

2×6

- State the Varignon's theorem.
- State the principle of parallelogram law of forces.
- State the necessary and sufficient conditions of equilibrium of a system of coplanar forces.
- A particle moves along a straight line under the law of motion given by $x = a \cos(nt + b)$. Show that the acceleration is directed to the origin and varies as the distance.
- If the tangential and normal components of acceleration of a particle be equal then find its velocity.
- Find the horse-power of an engine which can project 10,000 lbs. of water per minute with a velocity of 80 ft. per second.
- Show that in a central orbit, the angular momentum is always conserved.
- Define Apse in a central orbit of a particle.
- If the radial velocity of a particle is proportional to its cross-radial velocity, find its path in polar co-ordinates.
- Two bodies of masses m and $4m$ are moving with equal momentum. Find the ratio of their kinetic energy.

Group - B

2. Answer *any seven* questions :

6×7

- A system of coplanar forces has total moments H , $2H$ and $3H$ about the points $(0, 0)$, $(0, 1)$ and $(2, 4)$ respectively. Find the components of the resultant parallel to the coordinate axes and the equation of its line of action.
- Forces $\vec{P}$ and $\vec{Q}$, whose resultant is $\vec{R}$, act at a point O . If any transversal cuts the lines of action of the forces $\vec{P}$, $\vec{Q}$, $\vec{R}$ at the points L , M , N respectively, then show that

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$$\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON},$$

where P, Q, R are the magnitudes of the forces $\vec{P}, \vec{Q}, \vec{R}$ respectively.

- (c) A particle moves in a straight line from rest under an attractive force (or an acceleration)

$$\mu \times (\text{distance})^{-2}$$

directed towards a fixed point on the line, where μ is a constant. Show that if the initial distance is $2a$, then the distance will be a after a time

$$\left(\frac{\pi}{2} + 1\right) \left(\frac{a^3}{\mu}\right)^{1/2}$$

- (d) A particle is projected vertically upwards with a velocity V . Assuming that the resistance of air is kv per unit mass, where v is the velocity of the particle at any instant and k is a constant, show that the particle comes to rest at a height

$$\frac{V}{k} - \frac{g}{k^2} \log \left(1 + \frac{kV}{g}\right)$$

above the point of projection, g being the acceleration due to gravity, supposed constant.

- (e) If h be the height attained by a particle when projected with a velocity V from the earth's surface supposing its attraction constant, and H the corresponding height when the variation of gravity is taken into account, prove that $\frac{1}{h} - \frac{1}{H} = \frac{1}{r}$, where r is the radius of the earth.

- (f) A particle is projected in a resisting medium whose resistance is k times velocity and the initial velocity is V . Show that $v = Ve^{-kt}$ and $s = \frac{V(1 - e^{-kt})}{k}$.

- (g) A train of mass M lbs. is ascending a smooth incline of 1 in n and when the velocity of the train is v ft/sec, its acceleration is f ft/sec², prove that the effective horsepower of the engine is

$$\left\{ \frac{Mv(nf + g)}{550gn} \right\}.$$

- (h) A car of mass m starts from rest and moves on a level road under a constant frictional resistance, the engine working at a constant rate P . If the maximum speed is V and the speed u is attained after travelling a distance s in time t , show that

$$t = \frac{s}{V} + \frac{mu^2}{2P}.$$

- (i) A bullet of mass m moving with a velocity u , strikes a block of mass M , which is free to move in the direction of motion to the bullet and is embedded in it. Show that the loss of K.E. is

$$\frac{1}{2} \frac{mM}{(M+m)} u^2.$$

- (j) A particle falls from a height h upon a fixed horizontal plane. If e be the coefficient of restitution, show that the whole distance described before the particle has finished rebounding is

$$\left(\frac{1+e^2}{1-e^2} \right) h$$

and that the whole time taken is

$$\sqrt{\frac{2h}{g}} \left(\frac{1+e}{1-e} \right)$$

- (k) Define Conservative system of forces. For a particle moving in a plane curve under a conservative system of forces, prove that the sum of K.E. and P.E. is constant.

Group - C

3. Answer **any three** questions :

- (a) A particle is moving in a plane curve. Find the components of velocity and acceleration at any time along and perpendicular to the radius vector drawn from a fixed point in the plane. 3+4
- (b) Establish the differential equation

$$\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}$$

of the path for the motion of a particle describing a central orbit under an attractive force P per unit mass. 7

- (c) (i) State Kepler's Laws of planetary motion.
- (ii) An artificial satellite goes round the Earth in 90 minutes in a circular orbit. Calculate the height of the satellite above the earth, taking the earth to be a sphere of radius 6370 km and g at the orbit of the satellite to be 980 cm/sec². 3+4
- (d) If the angular velocity about origin be a constant ω , deduce the cross-radial component of rate of change of acceleration of the particle and show that if this rate of change of acceleration is zero,

$$\text{then } \frac{d^2r}{dt^2} = \frac{1}{3} \omega^2 r. \quad 7$$

- (e) A particle moves in a plane such that its accelerations parallel to the x and y axes are $K^2a \sin Kt$ and $K^2a \cos Kt$ respectively with the initial conditions $x = 0, y = -a, \frac{dy}{dt} = 0, \frac{dx}{dt} = -Ka$ when $t = 0$. Find the equation of the path. 7

- (f) A particle is describing an ellipse under a force to the centre; if v, v_1 and v_2 are the velocities at the ends of the latus rectum, major and minor axes respectively, prove that $v^2 v_2^2 = v_1^2 (2v_2^2 - v_1^2)$. 7