

2026

MATHEMATICS — MDC

Paper : CC-5

(Advanced Calculus)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**(Symbols have their usual meaning.)*

Group - A

(Marks : 20)

1. Answer **any four** questions :

- (a) Show that the sequence $a_n = \frac{2n+1}{3n+5}$, is bounded and monotone. Hence find its bounds.

1½+1½+2

- (b) Prove that a monotone increasing sequence of real numbers that is bounded above, is convergent to the least upper bound.

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- (c) Define limit of a sequence $\{S_n\}$ of real numbers.

Show that $\lim_{n \rightarrow \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.

2+3

- (d) Let $\{x_n\}$ be a sequence defined by

$$x_n = (-1)^n \left(2 + \frac{1}{n} \right).$$

- (i) Find bounds of $\{x_n\}$ if any.

- (ii) Test the convergence of $\{x_n\}$.

3+2

- (e) Test the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$, using the Comparison Test.

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- (f) Examine the convergence of the series

5

$$\frac{1}{3} + \left(\frac{2}{5} \right)^2 + \left(\frac{3}{7} \right)^3 + \dots$$

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- (g) Test the convergence of the infinite series

$$\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{1}{5} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{7} + \dots$$

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Group - B**(Marks : 25)**

2. Answer
- any five**
- questions :

- (a) Define continuity of a function at a point and on an interval. Examine the continuity of

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ 2, & x = 1 \end{cases}.$$

1+1+3

- (b) A function
- f
- is defined on
- $\mathbb{R}$
- by

$$f(x) = \begin{cases} \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Prove that f is not continuous at 0.

5

- (c) Let
- $f: \mathbb{R} \rightarrow \mathbb{R}$
- defined by

$$f(x) = \begin{cases} x & \text{for } x \in \mathbb{Q} \\ 1-x & \text{for } x \in \mathbb{R}-\mathbb{Q}. \end{cases}$$

Show that $f(x)$ is continuous only at $x = \frac{1}{2}$ [$\mathbb{R}$: Set of real numbers $\mathbb{Q}$: Set of rational numbers].

5

- (d) Prove that there exists
- $x \in \left(0, \frac{\pi}{2}\right)$
- such that
- $\cos x = x$
- .

5

- (e) State Lagrange's Mean Value Theorem and explain its geometrical interpretation. Verify the theorem for

$$f(x) = x^2 \text{ on the interval } [1, 3].$$

1+2+2

- (f) If
- $f: [a, b] \rightarrow \mathbb{R}$
- is continuous on
- $[a, b]$
- and differentiable on
- (a, b)
- , show that there exists some
- $c \in (a, b)$
- such that

$$f(b) - f(a) = cf'(c) \log \left(\frac{b}{a} \right).$$

5

- (g) State the Intermediate Value Theorem. Show that the equation
- $x^3 - x - 1 = 0$
- , has at least one real root in the interval
- $(1, 2)$
- .

2+3

(h) Prove that $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. 5

(i) Apply Maclaurin's theorem to the function $f(x) = (1+x)^4$ to deduce that

$$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4. \quad 5$$

Group - C

(Marks : 30)

3. Answer **any six** questions :

(a) Define the limit of a function of two variables. Examine whether $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$ exists. 2+3

(b) Let $f(x, y) = \begin{cases} x \sin \frac{1}{y} + \frac{x^2 - y^2}{x^2 + y^2}, & \text{if } y \neq 0, \\ 0, & \text{if } y = 0. \end{cases}$

Show that $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y)$ exists but neither $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ nor $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ exists. 5

(c) Give an example of a function $f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ such that f is continuous at $(0, 0)$, but not differentiable at $(0, 0)$. 5

(d) Let $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$ for all $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$, even though the conditions of Schwarz's theorem are not satisfied by f . 3+2

(e) If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$. 5

(f) For the function $f(x, y) = x^2 y + 3xy^2$, find

(i) The first-order partial derivatives,

(ii) The second-order mixed partial derivatives,

(iii) Verify that. 1+2+2

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}.$$

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- (g) Show that $y^2 - x^2y - 2x^5 = 0$ determines y uniquely as a function of x near the point $(1, -1)$ and find $\frac{dy}{dx}$ at $(1, -1)$. 5

- (h) State the chain rule for functions of two independent variables.

If $z = x^2 + y^2$, $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$. 1+2+2

- (i) Let $f(x, y) = \begin{cases} xy^2, & y > 0 \\ -xy^2, & y \leq 0 \end{cases}$.

Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$. 5

- (j) State and prove Euler's theorem for the function two variables. 2+3
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