

2026

## MATHEMATICS — MINOR

Paper : MN-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Symbols have their usual notation.*

## Group - A

1. Answer **any five** questions :

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- (a) Show that the product of all the values of  $(1+i\sqrt{3})^{\frac{3}{4}}$  is 8.
- (b) Let  $\alpha, \beta, \gamma$  be three roots of the equation  $x^3 + 2x^2 + 3x + 1 = 0$ . Find the equation whose roots are  $\alpha(\beta + \gamma), \beta(\gamma + \alpha), \gamma(\alpha + \beta)$ .
- (c) Using Descartes Rule of Sign, state possible number of positive, negative and imaginary roots of the equation  $x^5 + 2x^4 + 7x^3 + 9x^2 - 3x - 9 = 0$ .
- (d) If  $a, b, c, d$  are four positive numbers, prove that  $(a + b + c + d)(a^3 + b^3 + c^3 + d^3) \geq 16abcd$ .
- (e) If  $a + \frac{1}{a} = 2\cos\alpha$ , then prove that  $a^n + \frac{1}{a^n} = 2\cos n\alpha$ .
- (f) The product of two roots of the equation  $x^4 + x^3 + 2x^2 + 2x + 4 = 0$  is equal to the product of the other two roots. Find the roots.
- (g) Solve the equation  $x^3 - 12x + 65 = 0$  by Cardan's method.
- (h) Solve the equation by Ferrari's method  $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ .
- (i) Prove that  $\sin\left[i\log\frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$ , where  $a, b$  are real numbers not both zero.

## Group - B

2. Answer **any five** questions :

- (a) Examine if the relation
- $R$
- on the set
- $\mathbb{Z}$
- is an equivalence relation;

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 2a + 3b \text{ is divisible by } 5\}.$$

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- (b) Solve the following system of congruences :

$$x \equiv 2 \pmod{7}, x \equiv 5 \pmod{19}, x \equiv 4 \pmod{15}.$$

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- (c) Use mathematical induction to prove the sum of first  $n$  natural number is  $\frac{n(n+1)}{2}$ . 5
- (d) Let  $f: A \rightarrow B$  and  $g: B \rightarrow C$  be two mapping. If  $g \circ f: A \rightarrow C$  is bijective, prove that  $f$  is injective and  $g$  is surjective. 5
- (e) (i) Find all solution of  $5x \equiv 3 \pmod{19}$ .  
(ii) If  $n$  is a positive integer such that  $n^3 + 1$  is a prime number, show that  $n = 1$ . 3+2
- (f) Let  $A = \mathbb{R} \setminus \{3\}$  and  $B = \mathbb{R} \setminus \{1\}$ . Define  $f: A \rightarrow B$  by  $f(x) = \frac{x-2}{x-3}$ . Show that  $f$  is one to one and onto. 5
- (g) If  $p$  and  $p^2 + 8$  are both prime numbers, prove that  $p = 3$ . 5
- (h) A relation  $\rho$  is defined on  $\mathbb{Z}$  by  $x \rho y$  holds if and only if  $4x + 7y$  is divisible by 11 for  $x, y \in \mathbb{Z}$ . Show whether  $\rho$  is an equivalence relation on  $\mathbb{Z}$  or not. 5
- (i) Show that the functions  $f: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}$  and  $g: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{2\}$  defined by  $f(x) = \frac{x}{x-2}$  and  $g(x) = \frac{2x}{x-1}$  are inverses to each other. 5

### Group - C

3. Answer **any five** questions :

(a) Solve the following system of linear equation :

$$x_1 - 3x_3 = 8$$

$$2x_1 + 2x_2 + 9x_3 = 7$$

$$x_2 + 5x_3 = -2$$

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(b) Find the rank of the matrix by reducing it to row reduced echelon form

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

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(c) If  $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ , then show that  $A^2 - 4A - 5I = 0$ .

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- (d) Find a spanning set of  $\mathbb{R}^3$  containing the vectors  $(1, 2, 1)$  and  $(2, 1, 1)$ . 5
- (e) Find the dimension and basis of the solution space of the system of equations :  

$$x - 2y + z = 0$$

$$x - 2y - z = 0.$$
 5
- (f) For what values of  $h$  and  $k$ , the system of equations  

$$x_1 + 2x_2 + x_3 = 8, \quad 2x_1 + x_2 + 3x_3 = 13, \quad 3x_1 + 4x_2 - hx_3 = k$$
has (i) unique solution, (ii) no solution, (iii) an infinite number of solutions. 5
- (g) For what real values of  $k$ , the set  $S = \{(k, 1, k), (0, k, 1), (1, 1, 1)\}$  form a basis of  $\mathbb{R}^3$ . 5
- (h) (i) Find the value of  $\lambda$  so that the matrix  $A = \begin{pmatrix} 1 & 0 \\ 1 & \lambda \end{pmatrix}$  is its own inverse.
- (ii) If  $\alpha = (1, 1, 2)$ ,  $\beta = (0, 2, 1)$ ,  $\gamma = (2, 2, 4)$ , determine whether  $\beta$  is a linear combination of  $\gamma$  and  $\alpha$ . 3+2
- (i) If  $P = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$  and  $Q = \begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$ ; then find rank of  $P + Q$ . 5