

2026

PHYSICS — HONOURS

Paper : DSCC-5

(Modern Physics)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any five** questions from the rest.1. Answer **any five** questions :

3×5

- (a) A 45 kW broadcasting antenna emits radio waves at a frequency of 4 MHz. How many photons are emitted per second?
- (b) A relativistic electron has a de-Broglie wavelength of $0.15\text{\AA}$. Compute the phase and group velocities of the de-Broglie waves (rest mass of electron = $0.511\text{ MeV}/c^2$).
- (c) Which of the following functions can represent an acceptable wave function in the range $-\infty < x < \infty$? Give reasons for your answer.

(i) e^{ikx} (ii) e^{-x^2} (iii) $\sqrt{x}$ (iv) $\tan x$.

- (d) Compute the probability current density for a Gaussian wave packet $\psi(x) = Ae^{-\frac{x^2}{a^2} + \frac{i}{\hbar}p_0x}$.
- (e) Evaluate $[x, \sin p_x]$.
- (f) Prove that the expectation value of the square of any Hermitian operator is non-negative. Hence, show that the uncertainty associated with an observable can never be negative.
- (g) Write down momentum representation of position operator $\hat{x}$. Find the momentum space representation of the position space function $\psi(x) = \delta(x - a)$.
- (h) Find the eigenfunction of the operator $\left(\hat{x} + \frac{i}{\hbar}\hat{p}_x\right)$.

2. (a) Explain stopping potential and threshold frequency in photoelectric emission. Give an appropriate graph showing the variation of the stopping potential with the frequency of the incident radiation. Explain how Einstein's theory accounts for the variation.
- (b) If a retarding potential of 5V just stops the fastest photo-electrons emitted from cesium, what is the wavelength of the most energetic incident photons? Work function of cesium is 1.8 eV.

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- (c) A monochromatic X-ray beam is incident on two different target materials. Will the Compton wavelength shift be different for the two materials? Explain.
- (d) Explain why a plane de-Broglie wave can not represent a localized particle. How does the concept of a wave packet resolve this difficulty? (2+1+2)+3+2+2

3. A photon of energy E is incident on a free electron initially at rest. After scattering, the photon emerges with energy

$$E' = \frac{E}{1 + \alpha(1 - \cos \theta)},$$

$\left(\alpha = \frac{E}{m_e c^2} \right)$ at an angle θ with respect to its original direction. The electron recoils with kinetic energy

K and momentum $\vec{p}_e$, making an angle ϕ with the incident photon direction.

- (a) Write down the equations expressing the conservation of energy and momentum for the Compton scattering process.
- (b) Calculate K in terms of E , m_e and θ . Determine the maximum value of the recoil kinetic energy of electron.
- (c) Show that $\tan \phi = \frac{1}{1 + \alpha} \cot \frac{\theta}{2}$. 3+5+4
4. (a) Write the time dependent Schrödinger equation in one dimension. If $\psi(x, t) = \psi(x)\phi(t)$, then find the form of $\phi(t)$.
- (b) Define Hermitian operator in Quantum mechanics. Show that the operator $\frac{d}{dx}$ is anti-Hermitian for the class of wave functions which go to zero at infinity.
- (c) Find the Hermitian conjugate of the operator $\left(-\frac{d}{dx} \right)$.
- (d) Check the following operator is linear or not :

$$Af(x) = \hat{x}\hat{p}_x^2 f(x). \quad (1+3)+(1+3)+2+2$$

5. (a) Show that $(\hat{D} + \hat{x})(\hat{D} - \hat{x}) = \hat{D}^2 - \hat{x}^2 - 1$, where $\hat{D} = \frac{d}{dx}$.

- (b) Given $\hat{A} = \hat{p}_x - i\hat{x}^2$, $\hat{B} = \hat{p}_x + i\hat{x}^2$,

(i) Determine whether $\hat{A}$ and $\hat{B}$ Hermitian operators.

(ii) Find $[\hat{A}, \hat{B}]$. Is it Hermitian?

- (iii) Can the observables represented by $\hat{A}$ and $\hat{B}$ be measured simultaneously with arbitrary precision? Explain. 4+(2+3+1+2)
6. (a) The energy eigenvalue and the corresponding eigenfunction for a particle of mass m in a one dimensional potential $V(x)$ are $E = 0$ and $\psi(x) = \frac{A}{x^2 + a^2}$, respectively. Deduce the potential $V(x)$.
- (b) Show that the eigenvalues of a Hermitian operator are real.
- (c) Prove that two eigenfunctions of a Hermitian operator, belonging to different eigenvalues, are orthogonal to each other.
- (d) Two Hermitian operators $\hat{A}$ and $\hat{B}$ possess a common complete set of eigenfunctions. What is the least possible value of the product of uncertainties $\Delta A \Delta B$? Justify your answer. 3+3+3+(2+1)
7. (a) A particle is constrained to move on a path of length L . Obtain the normalized eigenfunctions of the momentum operator subject to the periodic boundary condition $\psi(x + L) = \psi(x)$ and show that the allowed momentum eigenvalues are quantized.
- (b) At some fixed time, a normalized state of the particle is given by

$$\psi(x) = \sqrt{\frac{2}{L}} \left(\frac{1}{\sqrt{3}} \sin \frac{2\pi x}{L} + \sqrt{\frac{2}{3}} \cos \frac{6\pi x}{L} \right).$$

Using the momentum eigenfunctions obtained in part (a), express $\psi(x)$ as a superposition of momentum eigenfunctions. Hence, determine all possible values of the momentum and the probability associated with each value.

- (c) If the expectation value of the momentum is $\langle \hat{p} \rangle$ for any wave function $\psi(x)$, then find the expectation value of the momentum for the wave function $e^{ikx}\psi(x)$. (2+1)+(2+2+2)+3
8. A particle of mass m moves in the one-dimensional finite rectangular potential well

$$V(x) = \begin{cases} 0, & x < 0, \\ -V_0, & 0 < x < a, \\ 0, & x > a, \end{cases}$$

where $V_0 > 0$. Assuming that the particle is in a bound state ($-V_0 < E < 0$)

- (a) Write down the time-independent Schrödinger equation in each region and obtain the wave function in the three regions.
- (b) State the boundary conditions satisfied by the wave function at $x = 0$ and $x = a$. Explain the physical reason for rejecting some of the solutions in the regions $x < 0$ and $x > a$.
- (c) Calculate the probability current density in each region and comment on your result.
- (d) Draw the ground and first excited state of wave functions. 5+3+2+2

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9. The ground state and first excited state wave functions of a one dimensional simple Harmonic oscillator

are $\psi_0(x) = \sqrt{\frac{\alpha}{\sqrt{\pi}}} e^{-\alpha^2 x^2 / 2}$ and $\psi_1(x) = \sqrt{\frac{2\alpha}{\sqrt{\pi}}} \alpha x e^{-\alpha^2 x^2 / 2}$ respectively, where $\alpha = \sqrt{m\omega/\hbar}$.

- (a) Is the ground state an eigenfunction of momentum operator?
 - (b) Determine the parity of the wave functions.
 - (c) Find the $\langle x \rangle$ for both the states.
 - (d) Find $\langle \hat{p} \rangle$ for the ground state. Can you explain the result physically? 2+2+(2+3)+(2+1)
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