

2026

PHYSICS — HONOURS

Paper : DSCC-7

(Mathematical Physics - II)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any five** questions from the rest.1. Answer **any five** questions :

3×5

- (a) Consider the differential equation $y'' - \frac{1+x}{x}y' + \frac{y}{x} = 0$. Find the Wronskian. Given one solution $y_1(x) = x$, find the other solution.
- (b) Starting from the generating function show that for legendre polynomial $P_l(-x) = (-1)^l P_l(x)$.
- (c) Show that the eigenvalues of a unitary matrix has unit modulus.
- (d) Find the eigenvalues and eigenvectors of the matrix, $A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.
- (e) Show that under an orthogonal transformation $\sum_{i,j=1}^3 A_{ij}A_{ij}$ is a scalar where A_{ij} is a second rank tensor.
- (f) Prove that $\text{Tr}(AB) = \text{Tr}(BA)$, where A, B are finite-dimensional $n \times n$ matrices.
- (g) Find the solution of the differential equation $x^2y'' + 4xy' + 2y = 0$.
- (h) Construct the matrix $P = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$.

2. (a) Consider the differential equation

$$x^2y'' + xy' + x^2y = 0.$$

- (i) Find the nature of the Singularity at $x = 0$.
- (ii) Find a solution of the equation about $x = 0$ using Frobenius method.

Please Turn Over

(7287)

- (b) Define $P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l$. Find $P_1(x)$ and $P_2(x)$. Sketch $P_1(x)$ and $P_2(x)$ in the same plot. (2+6)+(2+2)

3. Starting from the generating function of the Hermite polynomials $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)t^n}{n!}$.

- (a) Find $H_1(x)$ and $H_2(x)$.
 (b) Show that $H_1(x)$ and $H_2(x)$ are orthogonal.
 (c) Show that

(i) $H'_n(x) = 2nH_{n-1}(x)$

(ii) $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$.

4+2+(3+3)

4. (a) Show that the expansion coefficients C_i in $|\psi\rangle = \sum_i C_i |e_i\rangle$ are unique, where $\{|e_i\rangle\}$ is a basis set.
 (b) Show that a set of 2×2 matrices with complex elements is a vector space. Use matrix addition as vector addition.
 (c) The inner product of two vectors in two dimensions $|a\rangle$ and $|b\rangle$ is defined as $\langle a|b\rangle = a_1 b_1 - a_2 b_2$. Check whether it satisfies the properties of inner product.
 (d) For the vector

$$|\psi\rangle = 4|1\rangle + 3i|2\rangle,$$

where $\langle 1|1\rangle = \langle 2|2\rangle = 1$ and $\langle 1|2\rangle = 0$.

(i) Write $\langle\psi|$.

(ii) Normalize the vector $|\psi\rangle$.

3+3+3+(1+2)

5. (a) Find the eigenvalues and corresponding normalized eigenvectors for the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

- (b) Show that the trace of a matrix remains invariant under a similarity transformation.

- (c) Find the eigenvalues of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ without using characteristic equation. 6+3+3

6. (a) Consider two dimensional vectors $A_i = (a_1, a_2)$ and $B_j = (b_1, b_2)$. How many components does the tensor $A_i B_j$ have? Show that it can be decomposed as a sum of symmetric and antisymmetric tensor.
- (b) Show that $\vec{A} \cdot (\vec{B} \times \vec{C})$ is invariant under orthogonal transformation.
- (c) Find the Jacobian of the coordinate transformation from Cartesian to cylindrical polar coordinates.
(1+3)+5+3

7. Consider the following two vectors

$$|v_1\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; |v_2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

- (a) Calculate the inner product and determine whether they are orthogonal.
- (b) Check whether $|v_1\rangle$ and $|v_2\rangle$ are linearly independent. Hence determine whether they form a basis.
- (c) Construct an orthonormal set of vectors $|e_1\rangle$ and $|e_2\rangle$ from $|v_1\rangle$ and $|v_2\rangle$ using Gram-Schmidt orthogonalization procedure.
- (d) Verify that $|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| = I$, where I is the identity operator. 2+3+4+3
8. (a) Consider a function $u(x, y)$ defined on a two-dimensional grid with spacing Δx and Δy . Using Taylor expansion, derive the central difference approximations of the second partial derivative at a grid point (x_i, y_j) . Mention the order of accuracy of the obtained finite difference formulae. Write the central difference approximation for the mixed partial derivative $\frac{\partial^2 u}{\partial x \partial y}$ at the grid point (x_i, y_j) .
- (b) The temperature distribution in a thin metal plate is described by the function $T(x, y)$. The temperature values at different grid points are given below :

	$y - \Delta y$	y	$y + \Delta y$
$x - \Delta x$	95	100	105
x	98	102	106
$x + \Delta x$	101	104	108

where $\Delta x = \Delta y = 0.5$ cm.

Using finite difference approximations, calculate

- (i) The first partial derivative $\frac{\partial T}{\partial x}$ at the point (x, y) using the central difference formula.
- (ii) The second partial derivative $\frac{\partial^2 T}{\partial x^2}$ at the same point.
- (iii) Calculate $\nabla^2 T$ at the same point. (3+1+2)+(2+2+2)