

2026

PHYSICS — HONOURS

Paper : DSCC-8

(Classical Mechanics and Special Theory of Relativity)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer **question no. 1** and **any five** questions from the rest.1. Answer **any five** questions :

- (a) Consider a frame of reference K' moving with translational velocity $\vec{V}(t)$ relative to an inertial frame K . A particle is moving with velocity $\vec{u}(t)$ in K' . Find the fictitious force acting on the particle. Obtain the expression of potential energy that gives the fictitious force on the particle. 2+1
- (b) Write down parallel axes theorem and perpendicular axes theorem. 3
- (c) Write down the Lagrangian of a simple pendulum for small angle displacement θ from equilibrium position and find out the conjugate momentum p_θ . 1+2
- (d) Hamiltonian of a system is defined as $H = \sum_{i=1}^n p_i \dot{q}_i - L$. Find dH and identify the independent variables of the function H (symbols have their usual meanings). 1+2
- (e) What is a cyclic co-ordinate? Show that for each cyclic co-ordinate there exists a constant of motion. 1+2
- (f) State the postulates of special theory of relativity. 3
- (g) A muon at rest has lifetime 2×10^{-6} s. What is its lifetime when it travels with a velocity $\frac{3}{5}c$, where c is the speed of light in vacuum? 3
- (h) Define the kinetic energy of a relativistic free particle. Draw the kinetic energy versus speed diagram. 2+1

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2. (a) The derivative of a vector $\vec{A}$ in a fixed frame and a rotating frame are related as

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + (\vec{\omega} \times \vec{A}),$$

where symbols have their usual meanings.

If $\vec{\omega} = \omega \hat{i}$, then find out the time derivatives of $\hat{i}, \hat{j}$ and $\hat{k}$ with respect to rotating co-ordinate system. Identify which unit vector remain constant as in fixed co-ordinate system.

- (b) Let the origin of both fixed and rotating co-ordinates be the same and a particle is moving under force $\vec{F}$ in the fixed inertial frame. Using the time derivative relation given above, find out the equation of motion of the particle with respect to the rotating frame. Identify all fictitious forces appearing in the equation.
- (c) Show that the shape of surface of the water in a bucket will be paraboloid of revolution when the bucket spins with fixed angular speed, about a vertical axis. (3+1)+5+3
3. (a) For a rigid body rotating about fixed axis, find out the components of angular momentum in terms of the components of angular velocity. Write down the matrix relation of L_x, L_y, L_z with $\omega_x, \omega_y, \omega_z$. Identify moment of inertia tensor I_{ij} . Show that I_{ij} is symmetric.
- (b) Consider a rectangular lamina body of mass m , length l and breadth b . Origin of co-ordinate system is set at the centre of mass and X axis goes along breadth, Y axis goes along length and Z axis goes perpendicular to the plane of the lamina. Calculate principal moments of inertia and products of inertia. (4+1+1+1)+(3+2)
4. (a) Show that moment of inertia of a body about any arbitrary direction from a given point can not be larger than largest principal moment of inertia or smaller than smallest moment of inertia where three principal moments of inertia are measured at that given point about the principal axes.
- (b) Find the moment of inertia of a homogeneous solid cube of side a and mass M about any axis passing through its center.
- (c) Consider a free symmetrical top rotating about its centre of mass. Use the Euler's equation to obtain the equations of motion. Show that angular momentum rotates about the axis of the top with constant angular velocity. 3+3+(3+3)
5. (a) Set up the Lagrangian of a spherical pendulum. Hence, obtain the equations of motion.
- (b) Write down the Lagrangian of a one-dimensional harmonic oscillator. Find out generalised momentum and hence, the Hamiltonian of the harmonic oscillator from the Lagrangian. Hence determine the canonical equations of motion (Hamilton's equations of motion). (3+3)+(1+1+2+2)
6. (a) Show that under time independent holonomic constraints, kinetic energy T of a system of particles will be a homogeneous quadratic function of generalized velocities. Hence also show that
- $$\sum_{i=1}^n \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T$$
- and the Hamiltonian will be total energy $T + V$, where V is the potential energy, which is independent of $\dot{q}_i$.

- (b) Consider Lagrangian L of a N particle system. Show that total angular momentum of this system will be conserved if the Lagrangian remains invariant under rotation $\vec{r}_i \rightarrow \vec{r}_i + \Delta\phi \hat{n} \times \vec{r}_i$ (isotropy of space), here $\hat{n}\Delta\phi$ is an arbitrary small constant angle of rotation $\Delta\phi$ about an arbitrary axis directed along $\hat{n}$. (3+2+2)+5
7. A particle of mass m is moving in a plane under a central force given by: $\vec{F} = -k\vec{r}$.
- Find the potential $V(\vec{r})$ assuming $V(\vec{0}) = 0$.
 - Write down the Lagrangian of the particle. Find the cyclic coordinate and the corresponding conjugate momentum.
 - Hence, show that the motion takes place on a plane. Obtain the equations of motion.
 - Obtain the Hamiltonian and Hamilton's equations of motion. 2+(1+2)+(2+1)+(2+2)
8. (a) Let in an initial frame two events occur at an interval of 10^{-7} sec and be separated by a distance 40 meter. Show that it is a space-like separation. Find an inertial frame where these two events will occur simultaneously.
- (b) Establish the energy-momentum dispersion relation $E^2 = p^2 c^2 + m_0^2 c^4$.
- (c) What is a two-dimensional Minkowski diagram? Draw the world line of a photon and a massive particle in uniform motion along the positive X -axis in the Minkowski diagram with both passing through the event: $ct = 0, x = 0$. 4+4+4
9. (a) Consider two inertial frames S and S' where S' is moving w.r.t. S with a constant velocity v along the common $\frac{X^{x'}}{X - X'}$ direction. A rod of length l is placed at rest at the origin of S' making an angle of $\frac{\pi}{6}$ w.r.t. the X' axis. Find the value of v so that the rod appears to make an angle of $\frac{\pi}{3}$ w.r.t. the X axis in S .
- (b) Let S' frame is moving with uniform velocity V along X axis with respect to an inertial frame S keeping their axes parallel with S frame. A particle is moving with velocity $\vec{v} = \frac{d\vec{r}}{dt}$ with respect to S frame. Determine the components of velocity with respect to S' frame.
- (c) A particle of rest mass m_0 moving with speed v collides and sticks to a stationary particle of rest mass M_0 . Show that the speed of the composite particle is given by

$$\frac{\gamma m_0 v}{M_0 + \gamma m_0},$$

$$\text{where } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}.$$

4+4+4