

2026

STATISTICS — HONOURS

Paper : DSCC-6

(Probability – III)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*1. Answer **any five** questions :

2×5

- (a) Let
- X
- be a continuous random variable having cdf

$$F(x) = \begin{cases} 0, & x \leq 1 \\ 1 - x^{-\alpha}, & x > 1; \alpha > 0 \end{cases}$$

Find a measure of skewness based on quantiles when $\alpha = 1/2$ and hence comment on the skewness of the distribution.

- (b) State the general form of Chebyshev's inequality.
- (c) Suppose a fair die is rolled repeatedly under similar conditions. Let X denote the number of rollings required to get the face value 6 four times. Identify the distribution of X and write its pmf.
- (d) Let X be a continuous random variable having characteristic function

$$\phi_X(t) = \left(1 - \frac{it}{4}\right)^{-4}, \quad t \in \mathbb{R}, i^2 = -1.$$

Write the pdf of X .

- (e) Suppose X has cdf $F(x) = \alpha F_1(x) + (1 - \alpha)F_2(x)$, $0 < \alpha < 1$, where F_1 is the cdf of *Cauchy* ($\theta_1, 1$) and F_2 is the cdf of *N* ($\theta_2, 1$). What may be the choices of θ_1 and θ_2 such that the median of F is 2026, when $\alpha = 1/4$ and why?
- (f) Let ξ_q be the quantile of order q ($0 < q < 1$) of $N(\mu, \sigma^2)$. Express ξ_q in terms of the upper percentile point of the standard normal distribution.
- (g) Suppose $X \sim \text{Gamma}(n, \theta)$. Show that $P(X < n\theta) > 1/2$.
- (h) Suppose $X \sim \text{Cauchy}(0, 1)$. Then "MGF of $Y = \sin(X)$ always exists". Justify the statement.

Please Turn Over

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2. Answer **any four** questions :

- (a) Derive the MGF of a continuous uniform distribution with support (a, b) , $-\infty < a < b < \infty$. Suppose X is a continuous random variable such that $P(0 \leq X \leq 1) = 1$ and $P(x \leq X \leq y)$ is a function of $(y - x)$, $\forall 0 \leq x < y \leq 1$. Show that $X \sim U(0, 1)$. 2+3
- (b) Suppose $X \sim \text{Cauchy}(0, 1)$. Find the cdf of : (i) $Y = -X$, (ii) $Z = 1/X$. Hence identify the distributions. 2½+2½
- (c) Suppose $(X, Y) \sim \text{BN}(50, 60, 4, 9, \rho)$, $-1 < \rho < 1$ and $Z \sim \text{Gamma}(\alpha = 10, p = 5)$, independent of (X, Y) . Define $X_1 = X + \lambda Z$ and $X_2 = Y + \lambda Z$, for some real constant λ . Find the correlation coefficient of X_1 and X_2 . Also show that X_1 and X_2 are uncorrelated for some negative value of ρ . 3+2
- (d) When is a distribution said to be truncated? Suppose $X \sim \exp(\text{mean} = \theta)$ and let c be a positive constant. Find the mean and variance of the distribution truncated below at c . 1+4
- (e) Suppose X has a negative binomial distribution with parameters r and $p = 1 - q$. Show that the distribution of X approaches to $\text{Poisson}(\mu)$ as $r \rightarrow \infty$ and $p \rightarrow 1$ ($q \rightarrow 0$) but $rq = \mu$ is held fixed. 5
- (f) Describe Pareto's Law. Hence derive Pareto distribution. 2+3

3. Answer **any three** questions :

- (a) (i) Let X and Y be two independent random variables with pmf

$$P(X = k) = p(1 - p)^k, k = 0, 1, 2, \dots$$

$$P(Y = v) = \pi(1 - \pi)^v, v = 0, 1, 2, \dots$$

where $0 < p, \pi < 1$. Show that $Z = \min\{X, Y\}$ has pmf

$$P(Z = t) = \theta(1 - \theta)^t, t = 0, 1, 2, \dots$$

where $\theta = p + \pi - p\pi$. Also find the pmf of $X + Y$ when $p = \pi$, and find the conditional distribution of X given $X + Y$.

- (ii) If X and Y are jointly distributed random variables with means 0, variances 1 and correlation coefficient ρ , show that $E\left[\max\{X^2, Y^2\}\right] \leq 1 + \sqrt{1 - \rho^2}$. 10+5

- (b) (i) Suppose X follows a double exponential (Laplace) distribution with location μ and scale σ . Find the MGF of X and hence obtain the first four moments. Comment on the nature of the distribution.

- (ii) Suppose $X | \theta \sim \text{Bin}(n, \theta)$, $n \in \mathbb{N}$, $\theta \in (0, 1)$, and $\theta \sim \text{Beta}_1(\alpha, \beta)$, $\alpha, \beta > 0$.

Show that $E(\theta | X = x) = (x + \alpha) / (n + \alpha + \beta)$. 10+5

- (c) Suppose X is a continuous distribution with bounded support such that

$$X = \begin{cases} Y, & \text{if } W > 0 \\ Z, & \text{if } W \leq 0 \end{cases}$$

where Y , Z and W are independent random variables with

$$Y \sim \text{Beta}_1(2, 5), Z \sim \text{Beta}_1(5, 2), W \sim N(0, 1)$$

Find the pdf of X . Hence find γ_1 and γ_2 for the distribution and comment. Do you have the same conclusion if $W \sim N(1, 1)$ instead of $N(0, 1)$? 9+6

- (d) (i) Define the logistic distribution with parameters μ and σ . Find the interquartile range (IQR) for $\text{Logistic}(\mu, \sigma)$ distribution and compare it with that of the $N(\mu, \sigma^2)$ distribution (given that $\tau_{0.25} = +0.6745$).

- (ii) Prove that if $X \sim \text{Logistic}(0, 1)$, then for any $t > 0$,

$$P(X \geq t) \leq e^{-\lambda t} \frac{\pi \lambda}{\sin(\pi \lambda)} \text{ for any } \lambda \in (0, 1).$$

- (iii) Suppose $X \sim \text{Logistic}(0, 1)$ and $Y \sim \text{Laplace}(0, 2)$. Show that for any $t > 0$

$$P(X > t) \leq P(Y > t). \quad \text{5+5+5}$$

- (e) Write short notes on :

(i) Lognormal distribution

(ii) Weibull distribution. 8+7