

2026

STATISTICS — MDC

Paper : CC-5

(Descriptive Statistics III and Probability III)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*1. Answer **any five** questions :

2×5

- State the relationship between a total correlation coefficient r_{12} and a partial correlation coefficient $r_{12.3}$.
- If $X \sim \text{Poisson}(\lambda)$, write down its skewness (γ_1) and kurtosis (β_2) in terms of λ .
- Define the Probability Generating Function (PGF) of a discrete random variable X .
- Under what limiting conditions does a Hypergeometric distribution tend to a Binomial distribution?
- State the memoryless property of the Exponential distribution mathematically.
- Let X and Y be two independent random variables. Prove that $E(XY) = E(X)E(Y)$.
- Write down the Joint Probability Mass Function (p.m.f.) of a Trinomial distribution with parameters n , p_1 and p_2 .
- In a Bivariate Normal Distribution $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, what is the conditional distribution of Y given $X = x$?

2. Answer **any four** questions :

5×4

- Prove that the Partial Correlation Coefficient $r_{12.3}$ can be expressed as :

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}}.$$

- Find the Moment Generating Function (MGF) of a Geometric distribution with success probability p . Hence, find its mean.
- A random variable X follows a Continuous Uniform distribution over the interval $[a, b]$:
 - Derive its cumulative distribution function (c.d.f.).
 - Find its mean and variance.

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- (d) Let X follow a Lognormal distribution such that $\ln X \sim N(\mu, \sigma^2)$. Derive the r -th raw moment $E(X^r)$.
- (e) Show that for any two random variables X and Y , the conditional expectation satisfies the law of total expectation: $E(X) = E[E(X|Y)]$.
- (f) For a Bivariate Normal Distribution, prove that if the correlation coefficient $\rho = 0$, then the random variables X and Y are stochastically independent.

3. Answer **any three** questions :

- (a) (i) In a three-variate data setup on (X_1, X_2, X_3) , derive the least-squares regression equation of X_1 on X_2 and X_3 .
- (ii) Define the Multiple Correlation Coefficient $r_{1.23}$. Prove that

$$1 - r_{1.23}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2). \quad 8+7$$

- (b) (i) Derive the Moment Generating Function (MGF) of a Binomial distribution $Bin(n, p)$. Use it to find its mean, variance and the third central moment (μ_3).
- (ii) Show that the Poisson distribution can be viewed as a limiting form of the Negative Binomial distribution, by clearly stating the necessary assumptions. 8+7
- (c) (i) Let X follow a Gamma distribution with parameters α (shape) and β (scale). Find its MGF. Use it to determine its mean and variance.
- (ii) Define a Beta distribution of the first kind. Derive its mean and variance. Also, show that variance cannot exceed $1/4$. 7+8
- (d) (i) The joint probability density function (p.d.f.) of (X, Y) is given by :

$$f(x, y) = \begin{cases} c(x+y) & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

- I. Find the value of the constant c .
- II. Find the marginal distributions of X and Y .
- III. Check whether X and Y are independent.
- (ii) Prove that the correlation coefficient (ρ_{xy}) between two random variables X and Y lies between -1 and $+1$. (3+4+3)+5
- (e) (i) If (X_1, X_2, X_3) obeys a Trinomial distribution with parameters n, p_1, p_2 , derive :
- I. the marginal distribution of X_1 .
- II. the conditional distribution of X_2 given $X_1 = x_1$.
- III. the covariance between X_1 and X_2 .
- (ii) State the properties of the marginal and conditional density functions of a Bivariate Normal Distribution. Explain how the regression curves behave in this setup. (4+4+3)+4