

2026

MATHEMATICS — HONOURS

Paper : DSCC-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and Symbols have their usual meaning.*

Group - A

1. Answer **any two** questions :

2½×2

(a) Show that the sum of 99th powers of the roots of the equation $x^7 - 1 = 0$ is zero.(b) For positive real numbers a, b, c , find the least value of $(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.(c) If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$, then prove that $x_1 x_2 x_3 \dots \text{to } \infty = 1$.2. Answer **any four** questions :

5×4

(a) Show that the solutions of the equation $(1+x)^{2n} + (1-x)^{2n} = 0$ are

$$x = \pm i \tan \frac{(2k-1)\pi}{4n}, k = 1, 2, \dots, n.$$

(b) If $\tan^{-1}(x + iy) = u - iv$, where x, y, u, v are real numbers and $(x, y) \neq (0, \pm 1)$, then prove that $x^2 + y^2 + 2x \cot 2u = 1$ and $x^2 + y^2 + 1 + 2y \coth 2v = 0$.(c) If $a_1, a_2, \dots, a_n$ are n positive rational numbers and $S = a_1 + a_2 + \dots + a_n$, then prove that

$$\left(\frac{S}{a_1} - 1\right)^{a_1} \left(\frac{S}{a_2} - 1\right)^{a_2} \dots \left(\frac{S}{a_n} - 1\right)^{a_n} \leq (n-1)^S.$$

(d) Prove that $\sin \left[i \log \frac{a-ib}{a+ib} \right] = \frac{2ab}{a^2 + b^2}$, where a, b are real numbers not both zero.

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- (e) State Descartes' rule of signs regarding the number of positive roots of an equation with real coefficients. Apply this rule to show that the equation $x^4 + 12x - 5 = 0$ has two real roots and two non-real roots.
- (f) Solve the equation $x^3 - 18x - 35 = 0$, by Cardan's method.

Group - B

3. Answer **any two** questions :

2½×2

- (a) If $S = \{1, 2, 3, 4\}$, then write an equivalence relation on S and also write the partition corresponding to the equivalence relation.

- (b) Is the mapping $f: \mathbb{R} \rightarrow (-1, 1)$ defined by $f(x) = \frac{1}{1+x^2}$, $x \in \mathbb{R}$ a bijective mapping? Justify your answer.

$\mathbb{R}$ = the set of real numbers and

$$(-1, 1) = \{x \in \mathbb{R} / -1 < x < 1\}.$$

- (c) Prove that the product of any three consecutive integers is divisible by 6.

4. Answer **any four** questions :

- (a) (i) Let $f, g: A \rightarrow B$ and $h: B \rightarrow C$ be mappings. If $h \circ f = h \circ g$ and h is injective, then prove that $f = g$.

- (ii) Let $f: X \rightarrow Y$ be a function and $B \subseteq Y$. Prove that $f(f^{-1}(B)) \subseteq B$. 3+2

- (b) Prove that the equation $ax + by = c$ has integral solution if $\gcd(a, b)$ divides c where a, b, c are integers such that a and b are not both zero. 5

- (c) Solve by Chinese remainder theorem : $x \equiv 2 \pmod{7}$, $x \equiv 3 \pmod{9}$, $x \equiv 2 \pmod{11}$. 5

- (d) (i) Let ρ be an equivalence relation on a set S and $a, b \in S$. Show that $cl(a) = cl(b)$ if and only if $a \rho b$.

- (ii) Find $\phi(2400)$. 3+2

- (e) (i) If p is a prime number such that $p|ab$, for some positive integer a, b , then prove that either $p|a$ or $p|b$.

- (ii) Find the remainder when $1! + 2! + \dots + 50!$ is divided by 15. 2+3

- (f) If a and b are relatively primes, then $\gcd[(a+b), (a-b)] = 2$ or 1. 5

Group - C

5. Answer **any two** questions :

2½×2

(a) If $x \neq 0$ and $\text{rank} \begin{bmatrix} x & y & z \\ y & z & t \end{bmatrix} = 1$, prove that $x:y:z:t = 1:\lambda:\lambda^2:\lambda^3$ for some λ .

(b) Examine linear dependence of the set of vectors $\{(1, -1, 2, 4), (2, -1, 5, 7), (-1, 3, 1, -2)\}$ in $\mathbb{R}^4$.

(c) Show that the following system of equations has infinite number of solutions :

$$x + 4y + 2z = 1, 2x + 7y + 5z = 1, 4x + 14y + 10z = 2.$$

6. Answer **any four** questions :

(a) (i) Is the system of equations $x + y - z = 6$, $2x - 3y + z = 1$, $2x - 4y + 2z = 1$ solvable? If so, solve it.

(ii) If $(0, 2, a)$ can be expressed as $(0, 2, a) = b(1, 2, 2) + c(1, 0, -1)$, find the values of a, b, c .
3+2

(b) (i) If the sum of each row of an $n \times n$ non-singular matrix A is $k (\neq 0)$, then prove that the sum of each row of A^{-1} is $\frac{1}{k}$.

(ii) Find the rank of the matrix $\text{adj}(A)$, where A is a 4×4 matrix of rank two. 4+1

(c) Find the values of λ and μ for which the system of equations :

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

have (i) no solution

(ii) unique solution and

(iii) infinitely many solutions. 5

(d) Find the row reduced echelon form of $A = \begin{pmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{pmatrix}$ and hence find the rank of it. 5

(e) Solve the system of equations : 5

$$\begin{aligned} x + 2y + z - 3w &= 1 \\ 2x + 4y + 3z + w &= 3 \\ 3x + 6y + 4z - 2w &= 4. \end{aligned}$$

(f) Show that the set $\{(1, 1, 0), (1, 0, 1), (0, 1, 1), (1, 1, 1)\}$ spans $\mathbb{R}^3$ but not linearly independent in $\mathbb{R}^3$. 5