

2026

MATHEMATICS — MDC

Paper : CC-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.* $\mathbb{Z}$ -set of integers, $\mathbb{R}$ -set of real numbers.

Group – A

(Marks : 25)

1. Answer *any two* questions :

2½×2

(a) Prove that $\log(-1) = (2n+1)\pi i$.(b) For three positive real numbers x, y, z prove that

$$\frac{2}{x+y} + \frac{2}{y+z} + \frac{2}{z+x} \geq \frac{9}{x+y+z}$$

(c) Find the value of m , if the polynomial $4x^3 - 3x^2 + 2x + m$ is divisible by $x + 2$.(d) Find the value of $(1+i)^{\frac{1}{3}}$.2. Answer *any four* questions :

5×4

(a) Show that the product of all the values of $(1+i\sqrt{3})^{\frac{3}{4}}$ is 8.(b) If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 1 = 0$, find the equation whose roots are $\alpha(\beta + \gamma), \beta(\alpha + \gamma), \gamma(\alpha + \beta)$.(c) For any three distinct positive real numbers a, b, c with $a + b + c = 1$, prove that

$$8abc < (1-a)(1-b)(1-c) < \frac{8}{27}$$

(d) If $\alpha + i\beta = \cos(\theta + i\phi)$, then prove that

$$\frac{\alpha^2}{\cos^2 \theta} - \frac{\beta^2}{\sin^2 \theta} = 1 \quad \text{and} \quad \frac{\alpha^2}{\cosh^2 \phi} + \frac{\beta^2}{\sinh^2 \phi} = 1$$

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(7411)

- (e) Solve by Ferrari's method : $2x^4 + 6x^3 - 3x^2 + 2 = 0$.
 (f) Solve by Cardan's method : $x^3 - 6x - 9 = 0$.
 (g) Prove that the sum of 99th powers of the roots of the equation $x^5 - 1 = 0$ is zero.

Group – B**(Marks : 25)****3. Answer *any two* questions :**

2½×2

- (a) Give an example of a relation which is transitive, symmetric but not reflexive.
 (b) Show that the product of any three consecutive integers is divisible by 6.
 (c) List all integers x in the range $1 \leq x \leq 100$ that satisfy $x \equiv 7 \pmod{17}$.
 (d) If $f: A \rightarrow B$ and $g: B \rightarrow C$ be two mappings, such that $g \circ f$ is injective and f is surjective. Prove that g is injective.

4. Answer *any four* questions :

- (a) (i) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^3 - x$. Determine whether the function f is one-one and onto.
 (ii) If $f \circ g = f$, does it necessarily imply $g = \text{Identity mapping}$? Justify. 3+2
 (b) Find the least positive integer which leaves remainders 2, 3 and 4 when divided by 5, 7 and 9 respectively. 5
 (c) Let define a relation ρ on $\mathbb{Z}$ by $a\rho b \Leftrightarrow a - b$ is divisible by 7. Prove that ρ is an equivalence relation. Find all equivalence classes. 3+2
 (d) Find the remainder when $(37^{73} + 73^{37})$ is divided by 5. 5
 (e) If p and $(p^2 + 8)$ are both prime numbers then prove that $p = 3$. 5
 (f) If $a \equiv b \pmod{m}$, then show that $a^n \equiv b^n \pmod{m}$ for all positive integers n . Prove by an example that the converse is not true. 4+1
 (g) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$. Show that $g = f^{-1}$ and verify that $f \circ g = I_{\mathbb{R}}$ and $g \circ f = I_{\mathbb{R}}$. 2+2+1

Group – C**(Marks : 25)****5. Answer *any two* questions :**

- (a) If A is a 2×2 matrix satisfying $A^2 = 5A - 6I$, show that A is invertible and find A^{-1} . 1½+1
 (b) Show that $(1, 2, 1)$, $(2, 1, 1)$ and $(1, 1, 2)$ are linearly independent vectors. 2½

(c) Is the system of equations solvable?

2½

$$x + y + z = 4$$

$$2x + 5y - 2z = 3$$

$$x + 7y - 7z = 5$$

(d) If $\{v_1, v_2, v_3\}$ is linearly independent set, state with proper justification whether the set $\{v_1 + v_2, v_2 + v_3, v_3 + v_1\}$ is linearly independent or not.

2½

6. Answer **any four** questions :

(a) Determine rank of the matrix $A = \begin{bmatrix} 1 & 3 & 5 & -1 \\ 2 & 1 & -2 & 8 \\ 0 & 5 & 12 & -10 \end{bmatrix}$.

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(b) Investigate for what values of δ, μ the following equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \delta z = \mu$$

have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

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(c) Show that the set $B = \{(1,2,1), (0,1,0), (0,0,1)\}$ is a linearly independent set of $\mathbb{R}^3$. Express the vector $(2, 1, 3)$ as a linear combination of the vectors of B . Is this linear combination unique? Justify your answer.

2+2+1

(d) Obtain the Row Reduced Echelon form of the matrix

$$\begin{pmatrix} 1 & 3 & -2 & -3 \\ -2 & 1 & 4 & -1 \\ 3 & 2 & 1 & 5 \\ 1 & 3 & 5 & 4 \end{pmatrix}$$

Hence solve the following system of linear equations :

$$x + 3y - 2z = -3, -2x + y + 4z = -1, 3x + 2y + z = 5.$$

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(e) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$ then prove that $A^3 + A^2 - 8A = 9I$. Hence find A^{-1} .

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(f) Show that the set $S = \{(1,0,0), (1,1,0), (1,1,1), (0,1,0)\}$ spans the vector space $\mathbb{R}^3(\mathbb{R})$.

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(g) Let

$$A = \begin{bmatrix} 3 & 4 & -3 & 5 \\ 1 & 2 & -1 & 7 \\ 4 & 1 & 2 & 9 \\ 2 & -1 & 4 & k \end{bmatrix}$$

If the rank of a matrix A is 3, find the value of k .

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