

2026

MATHEMATICS — MINOR

Paper : MN-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Throughout the question paper $\mathbb{R}$ denotes the set of real numbers. Other symbols have their usual meaning.*

Group - A

(Marks - 25)

1. Answer **any five** questions :

(a) (i) Find the values of $(i-1)^{\frac{1}{5}}$.

(ii) If $a + \frac{1}{a} = 2\cos\alpha$, then prove that $a^n + \frac{1}{a^n} = 2\cos n\alpha$, where n is an integer. 2+3

(b) Prove that, $\sin\left[i \log \frac{a-ib}{a+ib}\right] = \frac{2ab}{a^2+b^2}$, where a, b are real numbers not both zero. 5

(c) (i) Find the principal values of $\text{Log}\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$.

(ii) Apply Descartes' rule of signs to find the number of real roots of the equation $x^6 - 1 = 0$. 3+2

(d) Solve, by Cardan's method : $x^3 - 12x + 65 = 0$. 5

(e) If one root of the equation $x^3 + px^2 + qx + r = 0$ is equal to the sum of the other two roots, then prove that $p^3 + 8r = 4pq$. 5

(f) Using Sturm's theorem, find the nature and positions of the roots of the equation $x^3 - 7x + 2 = 0$. 5

(g) Prove that $(a+b+c)^2 \geq 3(ab+bc+ca)$, where a, b, c are three real numbers. 5

(h) Show that $n^n > 1 \cdot 3 \cdot 5 \dots (2n-1)$, where n is a positive integer greater than 1. 5

(i) Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$, if two of the roots are in the ratio $3 : 5$. 5

Please Turn Over

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Group - B**(Marks - 25)**2. Answer **any five** questions :

(a) If two mappings $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$ and $g(x) = x - 2$, then verify the equality of $f \circ g$ and $g \circ f$. 5

(b) Prove or disprove : $\rho = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : 3a \rightarrow 4b \text{ is divisible by } 7\}$ is an equivalence relation on $\mathbb{Z}$. 5

(c) Show that the function

$f: \mathbb{R} - \{\sqrt{2}, -\sqrt{2}\} \rightarrow \mathbb{R}$ defined by,

$$f(x) = \frac{x}{x^2 - 2}$$

is surjective, but not bijective. 3+2

(d) Use Euclidean algorithm to find integers u and v such that

$$\gcd(252, 595) = 252u + 595v. \quad 5$$

(e) Solve for x using Chinese remainder theorem : 5

$$x \equiv 3 \pmod{5}$$

$$x \equiv 4 \pmod{7}$$

$$x \equiv 6 \pmod{9}.$$

(f) Prove that $3^{2n} - 8n - 1$ is divisible by 64 for all $n \in \mathbb{N}$. 5

(g) If both of p and $p^2 + 8$ are prime numbers, prove that $p = 3$. 5

(h) Prove that $\phi(3n) = 3\phi(n)$ if and only if 3 is divisor of n , where ϕ is the Euler's Phi function. 5

(i) (i) Prove that $11^{34} \equiv 2 \pmod{17}$.

(ii) If $\gcd(a, b) = 1$, prove that $\gcd(a + b, a) = 1$. 3+2

Group - C**(Marks - 25)**3. Answer **any five** questions :

(a) Find the rank of the matrix

$$\begin{pmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{pmatrix}$$

by using elementary row operation. 5

- (b) Investigate for what values of β and μ the following equations

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \beta z = \mu$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

2+2+1

- (c) If $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, find Rank $(A + B)$. 5

- (d) If (α, β, γ) is a linearly independent set of vectors in $\mathbb{R}^3$, then prove that $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also linearly independent. 5

- (e) Reduce the following matrix to a row reduced echelon matrix : 5

$$A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ 2 & 5 & 3 & 9 \\ 3 & 8 & 5 & 14 \end{pmatrix}$$

- (f) Find a Spanning set of the subset : $S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$. 5

- (g) (i) Determine the values of k so that the rank of the matrix $\begin{pmatrix} 1 & k \\ 2 & k^2 \end{pmatrix}$ is 1.

(ii) Prove or disprove : the set $\{(2, 3, 1), (2, 1, 3), (1, 1, 1)\}$ is linearly dependent in $\mathbb{R}^3$. 3+2

- (h) Express the equations :

$$x + 2y + z = 1$$

$$2x + 4y + 3z = 3$$

$$3x + 6y + 4z = 4$$

in the form $AX = B$ and solve.

2+3

- (i) (i) Find the value of λ so that the matrix $A = \begin{pmatrix} 1 & 0 \\ 1 & \lambda \end{pmatrix}$ is equal to its own inverse.

(ii) Express $(-1, 2, 4)$ as linear combination of $(-1, 2, 0)$, $(0, -1, 1)$ and $(3, -4, 2)$ over $\mathbb{R}^3$.

2+3